Three-dimensional MHD Simulations of Jets from Accretion Disks

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1. Relativistic velocity up to $\sim c$

The velocity is almost equal to the escape velocity of the central object.

Consistent with the MHD model (see, e.g., Shibata & Uchida 1986, Kudoh, Matsumoto, & Shibata 1998).

**GRS1915+105**

Mirabel & Rodriguez 1994
Rodriguez & Mirabel 1999
Basic Properties of the Jets (2)

2. The jets extend over kpc to Mpc, keeping its collimation.

The jets must be capable of exceptional stability.

How about the stability of MHD jet?
Motivation of Our Research

The mechanisms of the jet launching from the accretion disk and the collimation: Shibata & Uchida 1986, Matsumoto et al. 1996, Kudoh et al. 1998 (2.5-D axisymmetric simulations).


In our research, it is investigated whether the MHD jets launched from the accretions disk are stable in 3-D, by solving the interaction of the magnetic field and the accretion disk.
Basic Equations

Ideal MHD Equations

\[
\frac{\partial \rho}{\partial t} + \mathbf{v} \cdot \nabla \rho = -\rho \nabla \cdot \mathbf{v}
\]

\[
\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} = -\frac{1}{\rho} \nabla \left( p + \frac{B^2}{8\pi} \right) + \frac{1}{4\pi \rho} \mathbf{B} \cdot \nabla \mathbf{B} + g
\]

\[
\frac{\partial p}{\partial t} + \mathbf{v} \cdot \nabla p = -\gamma p \nabla \cdot \mathbf{v}
\]

\[
\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0
\]

\[
\mathbf{E} = -\mathbf{v} \times \mathbf{B}
\]

The calculation scheme is CIP-MOC-CT.

I developed the 3-D cylindrical code by myself. The number of grid points is \((N_r, N_\varphi, N_z) = (171, 32, 195)\).
CIP-MOC-CT Scheme

CIP: A kind of Semi-Lagrange method. Using the CIP for solving the hydro-part of the equations. 3rd order interpolation with the physical value and its derivative. Therefore, the time evolution of the derivatives is also calculated (see, e.g., Kudoh, Matsumoto, & Shibata 1999).

MOC: The accurate method solving the propagation of the liner Alfven waves.

CT: Solving the induction equation with the constraint of divB=0.
Initial Condition (1)

Accretion disk: an rotation disk in equilibrium with the point-mass gravity, centrifugal force, and the pressure gradient force.

Initial magnetic field: a vertical and uniform large-scale magnetic field. The ratio of the magnetic to gravitational energy, $E_{mg} = (V_{A0}^2 / V_{K0}^2)$, is the parameter for the initial magnetic field strength.

The typical value is $E_{mg} = 5.0 \times 10^{-4}$. The plasma- in the disk~200, in the corona~4.
Nonaxisymmetric perturbation: the amplitude is the 10% of the sound velocity at \((r,z) = (1.0,0.0)\), and with the form of

1. \(\sin 2\theta\) (sinusoidal)
2. random number between \(-1\) and \(1\) instead of sinusoidal function (random)

\(E_{mg}\): 8 parameters

24 runs in total (including the no perturbation cases).
Time Evolution

Axisymmetric

Sinusoidal

Random

On the x-z plane

3D movie
Nonaxisymmetric in the Jet

The slice on this plane.

The jets seem to have the non-axisymmetric structure with m=2 even in the random perturbation case.
Power Spectra in the Jet and Disk

(1)

Time evolution of the Fourier power spectra of the non-axisymmetric modes of the magnetic energy.

\[ \tilde{E}_M (k_r, m, k_z) = \frac{1}{V_s} \int \int \int_{V_s} E_M (r, \phi, z) e^{i(k_r r + m\phi + k_z z)} r dr d\phi dz \]

Then, integrate about \( k_r, k_z \).
Power Spectra in the Jet and Disk

Almost constant levels (no growth).

The flare-up of the $m=2$ mode spectrum in the disk before the dominance of the $m=2$ mode in the jet.
Growth Rate of Nonaxisymmetric Modes of MRI

MRI: Magneto-Rotational Instability

\[
\frac{k^2}{k_z^2} \left[ \frac{d^2}{dt^2} + (k \cdot v_A)^2 \right] \delta B_R + \left[ \kappa^2 + 6 \frac{m^2}{k_z^2 R^2} \left( \frac{d\Omega}{d\ln R} \right)^2 \right] \left[ \frac{d^2}{dt^2} + (k \cdot v_A)^2 \right] \delta B_R \\
- \left[ 1 + \frac{m^2}{k_z^2 R^2} \left( \frac{d\ln \Omega}{d\ln R} \right)^2 \right] 4\Omega^2 (k \cdot v_A)^2 \delta B_R - 6 \frac{m k_R}{k_z^2 R} \frac{d\Omega}{d\ln R} \left[ \frac{d^2}{dt^2} + (k \cdot v_A)^2 \right] \frac{d \delta B_R}{dt} = 0
\]

Balbus & Hawley 1992, Eq. (2.24)

Solving this dispersion relation numerically, the growth rate of the m=2 mode is \( \gamma = 0.54 \) (detailed parameters).

\[ \exp[ \gamma t] = 5.1 \text{ (t=3.0) } \] On the other hand, the power spectrum of the m=2 mode became 5.9 times larger than the initial value (reference).
Amplification of the Magnetic Field in the Disk (1)

MRI □ Amplification of the magnetic energy
Check differences between the models.

Outer region: No significant difference among the models.
Inner region: Significant difference between the models.
Amplification of the Magnetic Field in the Disk (2)

\[ \frac{\partial}{\partial t} \left( \frac{B^2}{8\pi} \right) = -\mathbf{v} \cdot (\mathbf{J} \times \mathbf{B}) - \frac{1}{4\pi} \nabla \cdot (\mathbf{E} \times \mathbf{B}) \]

The work done by the Lorentz force.

Poynting flux

\[ \frac{\partial}{\partial t} \left( \frac{B^2}{8\pi} \right) = -\langle \mathbf{v} \cdot (\mathbf{J} \times \mathbf{B}) \rangle - \frac{1}{4\pi} \int (\mathbf{E} \times \mathbf{B}) \cdot d\mathbf{S} \]

\(<>\): Volume integral

Color function
Amplification of the Magnetic Field in the Disk (3)

The minus sign: kinetic $\rightarrow$ magnetic energy

The sum of the time integration of $\Box$ and $\Box$ = increase of $E_{mg}$

$\Box$ The difference between models A6 and S6 is consistent.
Not consistent between A6 and R6 $\Box$ Numerical Reconnection.
Angular Momentum Transport (1)

The mass accretion is important for the activity of AGNs, not limited to the jet formation.

How does it extract the angular momentum of the disk?

-disk model: assumption of the viscosity parameter.

Recently, it has been cleared that the magnetic turbulence is the origin of the viscosity.

How large is the amount of the extracted angular momentum in the radial direction? How about in the axial (z) direction?
Over a wide range of $E_{mg}$, the efficiencies of the angular momentum transport in the radial and axial directions are comparable.

The symbol “<< >>” means the spatial and temporal average.

### Angular Momentum Transport (2)

<table>
<thead>
<tr>
<th>$E_{mg}$</th>
<th>$\langle \frac{B_r B_\phi}{4\pi} \rangle / \langle p \rangle$</th>
<th>$\langle \frac{B_\phi B_z}{4\pi} \rangle / \langle p \rangle$</th>
<th>$\langle \frac{B_r B_\phi}{4\pi} \rangle / \langle p \rangle$</th>
<th>$\langle \frac{B_\phi B_z}{4\pi} \rangle / \langle p \rangle$</th>
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<tbody>
<tr>
<td>$1.0 \times 10^{-5}$</td>
<td>0.0035</td>
<td>0.0068</td>
<td>0.0041</td>
<td>0.0056</td>
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<tr>
<td>$2.0 \times 10^{-5}$</td>
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<td>0.012</td>
<td>0.0090</td>
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<td>$5.0 \times 10^{-5}$</td>
<td>0.018</td>
<td>0.022</td>
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<tr>
<td>$5.0 \times 10^{-4}$</td>
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<td>0.063</td>
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<tr>
<td>$1.0 \times 10^{-3}$</td>
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<td>0.082</td>
<td>0.12</td>
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<td>$2.0 \times 10^{-3}$</td>
<td>0.17</td>
<td>0.093</td>
<td>0.14</td>
<td>0.10</td>
</tr>
</tbody>
</table>

**Axisymmetric**  **Random**
Comparison with Steady Theory and Nonsteady Axisymmetric Simulation

\[ V_{\text{max}} \propto E_{\text{mag}}^{1/4} \]

\[ \frac{dM}{dt} \propto E_{\text{mag}}^{1/2} \]

\[ V_{\text{max}} \propto (E_{\text{mag}}/dM/dt)^{1/3} \]

\[ \frac{dM}{dt} \propto E_{\text{mag}}^{1/4} \]

Steady theory
Summary (1)

1. The jet launched from the accretion disk is stable, at least for 2.5 orbital periods of the accretion disk (there is no indication for the disturbance to grow).

2. The nonaxisymmetric disturbance made in the accretion disk owing to magnetorotational instability (MRI) propagates into the jet.

3. It is suggested that, in the random perturbation case, the magnetic field is complexly twisted and the numerical reconnection takes place in the inner region of the disk. We need to perform the resistive simulation in the future.
3. The efficiency of the angular momentum transport does not depend on the model (the type of the initial perturbation). The efficiencies in the radial (r) and axial (z) direction are comparable in the wide range of initial magnetic field strength.

4. Though the jet has the nonaxisymmetric structure, the macroscopic properties (e.g., the maximum jet velocity) are almost the same as those in the axisymmetric case shown by Kudoh et al. (1998).
Parameters for Solving the Dispersion Relation

Alfven velocity: $V_A = 0.056$ from the initial condition.

Radial wavelength: $\lambda_r = 0.4$ from the spatial distribution of $E_{mg}$.

Radial position: $R = 1.0$

Axial wavelength: $\lambda_z = 0.35$ ($\sim 2 \frac{V_A}{\Omega}$: most unstable).

Angular velocity: $\Omega = 1.0$ (angular velocity at $R = 1.0$)

Epicyclic frequency: $\nu = 0.0$ (constant angular momentum disk).
Growth of the Spectrum in the Disk

Increase by the factor of 5.9.

Linear growth  Nonlinear growth
Color Function

Color function $\Theta$. Initially,

$$\Theta = \begin{cases} 
1 & \text{inside of the disk} \\
0 & \text{outside of the disk} 
\end{cases}$$

Calculating the time evolution of $\Theta$ by

$$\frac{d\Theta}{dt} = \frac{\partial \Theta}{\partial t} + v_r \frac{\partial \Theta}{\partial r} + \frac{v_\phi}{r} \frac{\partial \Theta}{\partial \phi} + v_z \frac{\partial \Theta}{\partial z} = 0$$

The region where $\Theta$ is not equal to zero is the extent to which the matter originally in the disk reaches.
Steady Theory (1)

\[ v_\infty \approx \frac{B}{\sqrt{4\pi \rho}} \]

The terminal velocity of the jet is comparable to the Alfven velocity (magnetically accelerated).

\[ \frac{v_\varphi - r\Omega}{v_p} = \frac{B_\varphi}{B_p} \]

Seen from the corotating frame with the magnetic field, the velocity and magnetic fields are parallel (frozen-in condition).
Steady Theory (2)

At the infinity \( r \sim \infty \), \( V_\infty \sim 0 \) because the angular momentum is finite.

\[
\frac{r \Omega}{v_\infty} = \frac{B_\varphi}{B_p} \sim \square : \frac{B_\varphi}{B_p} \gg 1 \quad \longrightarrow \quad v_\infty \approx \frac{B_\varphi}{\sqrt{4 \pi \rho}}
\]

The mass outflow rate is expressed as \( \dot{M} = 4\pi r^2 \rho v_\infty \)

\[
v_\infty^2 = \frac{r^2 v_\infty B_\varphi^2}{\dot{M}} = \frac{r^4 \Omega^2 B_p^2}{\dot{M}} \quad \longrightarrow \quad v_\infty = \left( \frac{\Omega^2 B_p^2 r^4}{\dot{M}} \right)^{1/3}
\]
Steady Theory (3)

\[
\dot{M} \propto \begin{cases} 
B_p^0 & : \text{Strong initial magnetic field case (} B \sim B_p \gg B_\parallel \text{).} \\
B_p^1 & : \text{Weak initial magnetic field case (} B \sim B_\parallel \gg B_p \text{).}
\end{cases}
\]

See, e.g., Kudoh & Shibata 1995

Eventually,

\[
V_\infty \propto \begin{cases} 
B_p^{2/3} & \\
B_p^{1/3}
\end{cases}
\]

Maximum jet velocity (Kudoh et al. 1998).

\[V_z \propto E_{mg}^{1/6} \cdot B_p^{1/3}\]
Steady Theory (4)

Steady theory

\[ \dot{M} \propto \begin{cases} B_p^0 & \text{Nonsteady Simulations} \\ B_p^1 & \end{cases} \]

\[ \nu_\infty = \left( \frac{\Omega^2 B_p^2 r^4}{M} \right)^{1/3} \]

Michel’s solution

Mass accretion rate (Kudoh et al. 1998)

\[ \frac{dM_w}{dt} \propto E_{mg}^{1/2} \propto B_p^1 \]

Retrun