Astrophysical Turbulence

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- What is turbulence?
- What has been studied about turbulence?
- What are problems involving turbulence?
What is turbulence?

Turbulence is a flow regime characterized by high momentum convection, low momentum diffusion, and pressure and velocity variation with time.

The Reynolds number characterizes whether flow conditions lead to turbulence or not.

\[ \frac{\partial \vec{v}}{\partial t} = -\left( \vec{v} \cdot \vec{\nabla} \right) \vec{v} + \nu \nabla^2 \vec{v} - \frac{1}{\rho} \vec{\nabla} p \]

\[ \frac{\nu^2}{L} \quad \frac{\nu \nu}{L^2} \]

\[ \text{Re} \sim \frac{\nu^2}{L} \frac{\nu \nu}{L^2} \sim \frac{\nu L}{\nu} > \sim 100 - 1,000 \quad \rightarrow \text{turbulent!} \]
Van Dyke 1982 experiment

Edgar et al numerical simulations

Re = 24.8

Re = 0.16

Re = 9.6

Re ~ 1,500

Re = 140

Re >> 1

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turbulent flow around an obstacle; the flow further away is laminar

turbulence creating a vortex on an airplane wing

terrestrial examples 1
terrestrial examples 2
Astrophysical examples 1

Re >> 1 in astrophysical environments

A Solar filament

Jupiter's Great Red Spot from Voyager
astrophysical examples 2

Crab Nebula - supernova remnant

NGC 6302; Big, Bright, Bug Nebula - planetary nebula
Statistical description of turbulence

Power spectrum, $P_k$ - the portion of a signal’s power (energy per unit wavenumber) falling within given wavenumber

$\nu (\vec{r}), \rho (\vec{r}), \sqrt{\rho (\vec{r}) \nu (\vec{r})}, B(\vec{r}), ...$

$q(\vec{k}) \sim \int q(\vec{r}) d^3r$

Fourier transformation

$P_k \sim |q(\vec{k})|^2 k^2$

$\int P_k dk \sim \langle q(\vec{r}) \rangle^2$
Theory of turbulence

Kolmogorov's theory for incompressible hydrodynamic turbulence: it is based on the notion that that large eddies can feed energy to the smaller eddies and these in turn feed still smaller eddies, resulting in a cascade of energy from the largest eddies to the smallest ones.

On dimensional grounds, the only way of writing \( \varepsilon \) (energy transfer rate) in terms of \( V \) (velocity) and \( l \) (scale) is

\[
\varepsilon \sim \frac{V^2}{t} \sim \frac{V^3}{l} \sim \text{constant}
\]

\[
V \sim l^{1/3}
\]

\[
P_k \sim k^{-5/3}
\]

The spectrum of Kolmogorov turbulence
In astrophysical environments

\[ \text{Re} \sim \frac{\nu L}{\nu} \gg 1 \]

magnetic field exists in astrophysical environments:
- with magnetic field
- fluid \( \rightarrow \) drags magnetic field
- magnetic field \( \rightarrow \) exerts tension and pressure
- fluid and magnetic field moves together ("frozen")

\[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} + \frac{1}{\rho} \nabla p = \frac{1}{4\pi\rho} \left( \nabla \times \mathbf{B} \right) \times \mathbf{B} \]

\[ \frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left( \mathbf{B} \times \mathbf{v} \right) \]
with weak regular field (\( B_o \) small or \( v_A = \frac{B_o}{\sqrt{4\pi\rho}} \ll v \)) and still incompressible or subsonic super-Alfvenic turbulence

\[
M_A = \frac{v}{v_A} \gg 1
\]

more kinetic energy on larger scales, more magnetic energy on smaller scales, but \( P_k \propto k^{-5/3} \) for kinetic + magnetic power spectrum

(Haugen, Brandenburg et al, ...)
Goldreich & Sridhar model

for strong regular field

\[ ( B_o \text{ large or } v_A = \frac{B_o}{\sqrt{4\pi\rho}} \sim \nu ) \]

but still incompressible or subsonic


Applicable to most part of the ISM
Goldreich & Sridhar model

critical balance

\[
\frac{l_\perp}{l_{||}} \sim \frac{b_l}{B_0}
\]

constant energy cascade

\[
\mathcal{E}_{\text{cascade}} = \frac{b_l^2}{b_l / l_\perp} = \text{constant}
\]

\[
b \sim l_\perp^{1/3} \quad \text{or} \quad P_k \sim k^{-5/3}
\]

\[
l_{||} \sim l_\perp^{2/3}
\]
what the Goldreich & Sridhar model says

\[ \nu_\perp \sim l_\perp^{1/3} \]

\[ l_\parallel \sim l_\perp^{2/3} \]

Kolmogorov

larger anisotropy at smaller scales
but astrophysical turbulence is highly compressible!

\[ \frac{\delta \rho}{\rho} \gg 1 \]

and often highly supersonic!

\[ M_s = \frac{\nu}{c_s} \gg 1 \]

so astrophysical turbulence has to be studied numerically!
compressible hydrodynamic turbulence

sound mode (compressible mode)
  sound waves or shock waves
+ advection (incompressible or solenoid mode)
  mixing

hydrodynamics with the isothermal TVD code
3-D with $512^3$ and $256^3$ grid zones for various $M_s$

(Kim & Ryu 2005)
In 3D, there are both compressible and solenoidal modes. The slope changes from $-5/3$ to $-2$ as $M_s$ increases. The velocity power spectrum from 3D hydro simulations is given by $P_k \sim k^{-2}$. 

\[ P(k) \]
density power spectrum from 3D hydro simulations

In 3D, there are both compressible and solenoidal modes.

Slope changes from $-5/3$ to 0 as $M_s$ increases.
saw-toothed
distributions

3d hydro
turbulence
with $M_s = 1.2$

“saw-toothed”
distributions
3d hydro turbulence with $M_s = 12$

“peaked” distribution for density
“saw-toothed” distribution for velocity
compressible magnetohydrodynamic (MHD) turbulence

Alfven mode ($v = v_A \cos \theta$)
incompressible, restoring force=mag. tension

slow mode ($v \sim c_s$)
for magnetically dominated plasma ($v_A \gg c_s$), this is a sound wave along magnetic field; compression of gas

fast mode ($v \sim v_A$)
for magnetically dominated plasma ($v_A \gg c_s$), this is magnetic field compression wave; compression of B field
scaling relation for low $\beta$ and high $M_s$ turbulence

\[ \beta = \frac{p_{\text{gas}}}{p_{\text{magnetic}}} \]

Alfven $\sim k^{-5/3}$

slow $\sim k^{-5/3}$

fast $\sim k^{-3/2}$?

(Cho & Lazarian 2002)
what seems to have learned about compressible turbulence with low $\beta$ and high $M_s$ (applied to the ISM)

power spectra of velocity and magnetic field
- Alfven mode: Kolmogorov slop, anisotropic - G-S model
- slow mode: Kolmogorov slop, anisotropic (passively) - G-S model
- fast mode: -1.5 slop (?), isotropic

velocity power spectrum
- Alfven mode $>$ fast+slow mode
  (solenoid mode $>$ compressible mode)

density power spectrum
- shallower slop
Armstrong & Spangler (1995)

composite power spectrum from observations of various observations

power spectrum of electron column density in the interstellar medium

Kolmogorov slop

Armstrong & Spangler (1995)
density power spectrum of cold HI gas ($M_s \sim 2-3$)

dash line represents a dirty PS obtained after averaging the PW of 11 channels.
solid line represents a true PS obtained after cleaning.
much shallower power spectrum!

(Deshpande et al. 2000)
power spectra of various observed quantities: seem to be compatible with that of Kolmogorov turbulence in most observations, but not in all observations in astrophysical turbulence

— compressibility is important, or flow is supersonic
— magnetic field exists
— observed power spectrum is not that of velocity
Two star formation theories

SF regulated by AD

SF regulated by turbulence

ion
neutral
SPH calculations

sink particles $\rho > 10^4 \rho_0$

SFEs measured in 3D driven HD turbulent flows

- $M_*$ is mass fraction in sink particles.
- $\langle M_J \rangle_{turb}$ is effective turbulent Jeans Mass
- SFEs are very high in 3D driven HD turbulent flows, except cases driven at small scales.

(Klessen et al. 2000)

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Turbulence in the Coma Cluster ICM

(Schuecker et al. 2004)
Pressure fluctuations

Histogram of projected pressure fluctuations

$\delta \rho$ vs $\delta T$

Fluctuations are mostly gaussian and adiabatic

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noise subtracted power spectrum of projected pressure fluctuations with slope $n \sim -7/3 \ldots -5/3$

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energy content of turbulence as fraction of thermal energy

→ subsonic turbulent

close to Kolmogorov
Thank you!