3D Radiative MHD Simulation of Accretion Flow

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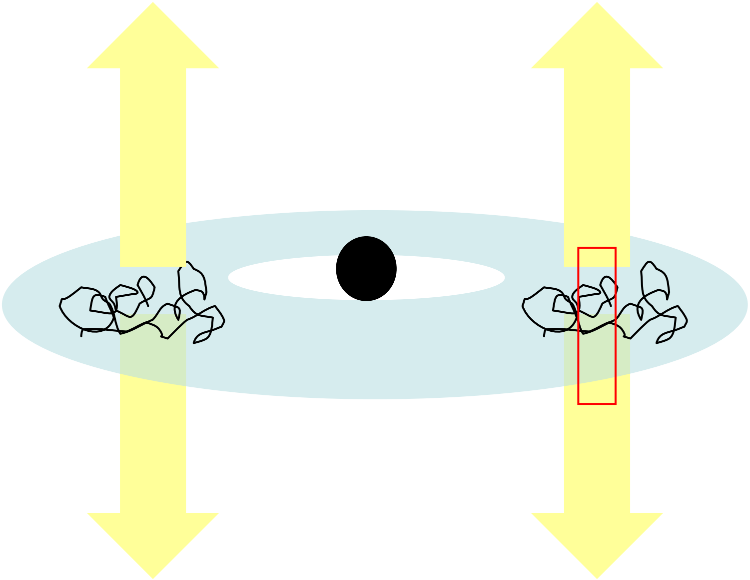


Standard model (α model) for geometrically thin and optically thick disk

Shakura & Sunyaev (1973)

- hydrostatic balance in the vertical direction: $dP/dz = -\rho\Omega^2 z$
- **viscosity: $T_{r\phi} = \alpha P$**
- local energy balance: $Q^{+}_{vis}(r) = Q^{-}_{rad}(r)$

(H, Σ, ρ, …) are functions of radius r for given parameters (M, M., and α)

one zone model

(a) inner region: $P \sim P_{radiation}$, $\chi \sim \chi_{\text{Thomson scattering}}$ … Radiation-dominated

(b) middle region: $P \sim P_{gas}$, $\chi \sim \chi_{\text{Thomson scattering}}$ … Gas-dominated

(c) outer region: $P \sim P_{gas}$, $\chi = \chi_{\text{free-free}}$

The α model fails in some problems …

e.g.

Radiation-dominated disk is unstable for

- viscous instability (Lightman & Eardley 1974)
- convective instability (Bisnovatyi-Kogan & Blinnikov 1977)
- thermal instability (Shakura & Sunyaev 1976)

MRI as a candidate for viscosity

Ab initio calculation is now possible for accretion disk.

e.g.

- Miller & Stone (2000) for gas-dominated disk (iso-thermal)
- Turner (2004) for radiation-dominated disk (FLD)

Purpose of this work is …

to obtain (stable) vertical structure of

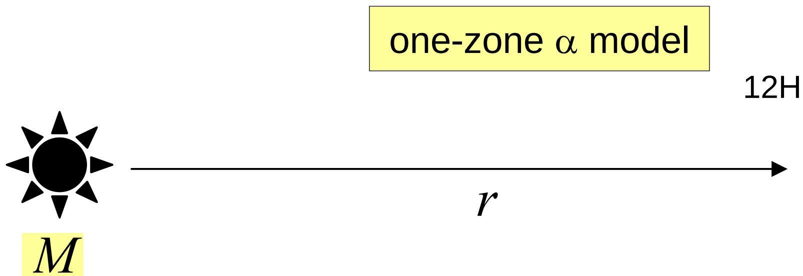
MRI-driven {radiation, gas}-dominated accretion disk,

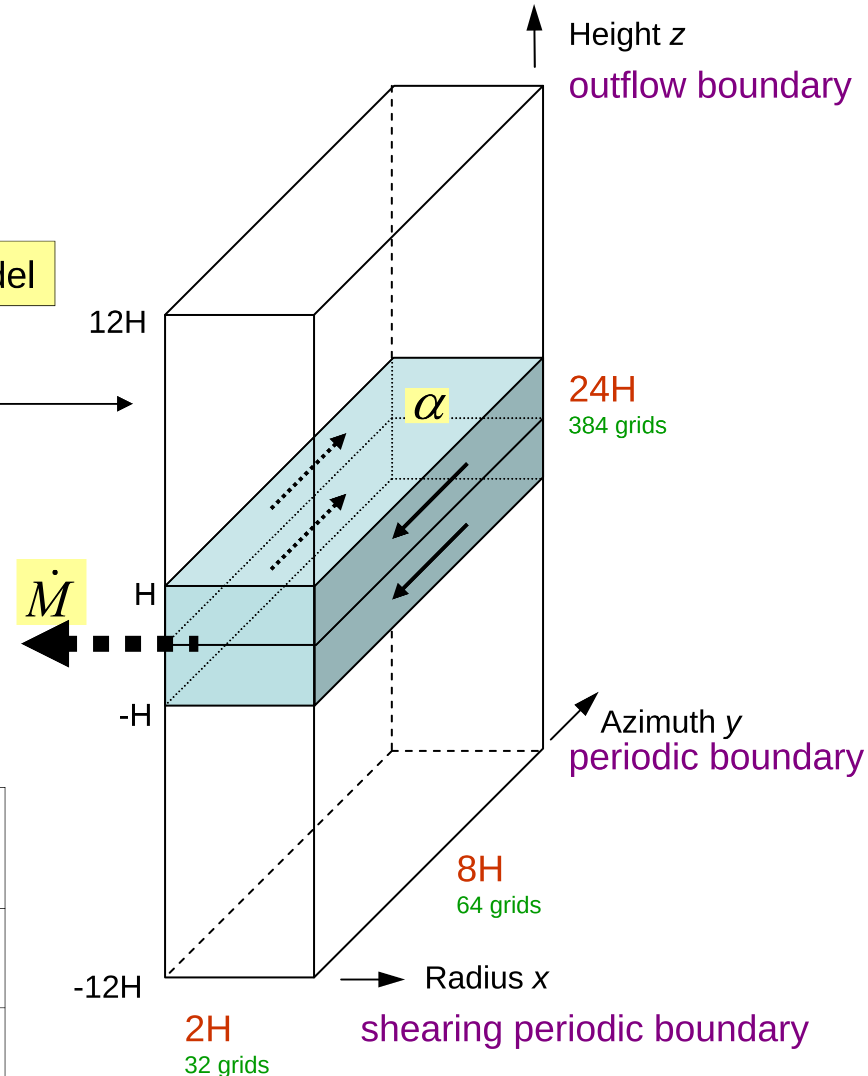
$\rho(z)$, $P(z)$, $E(z)$, $B(z)$, $v(z)$ for fixed radius,

using 3D FLD Radiation MHD simulation with local shearing box approximation.

Simulation Domain and Boundary Conditions

Local shearing box approximation is used to simulate a small patch of the disk, taking into account of vertical stratification.





parameters for initial condition

	M/M_*	$\dot{M}/\dot{M}_{Edd}$	α	r/r_g
Radiation dominated	10^8	0.1	0.01	200
Gas dominated	6.62	0.1	0.03	300

Basic Equations

3D equations of radiation MHD in the flux limited diffusion (FLD) approximation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$\frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho v v) = -\nabla p + j \times B + \frac{\bar{\chi}_{\text{Rosseland}} \rho}{c} F - 2\rho \Omega \times v + 3\rho \Omega^2 x - \rho \Omega^2 z$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (E v) = -\nabla v : \mathrm{P} + \bar{\kappa}_{\text{Planck}} \rho (4\pi B_{\text{Planck}} - cE) - \nabla \cdot F$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e v) = -(\nabla \cdot v) p - \bar{\kappa}_{\text{Planck}} \rho (4\pi B_{\text{Planck}} - cE)$$

$$\frac{\partial B}{\partial t} - \nabla \times (v \times B) = 0$$

$$F = -\frac{c\lambda}{\bar{\chi}_{\text{Rosseland}} \rho} \nabla E$$

$$p = (\gamma - 1) e$$

$$\mathrm{P} = \mathrm{f} \ E$$

- LTE: source function = Planck Function B_{Planck}
- energy-mean opacity = Planck-mean opacity: $\kappa_E = \kappa_{\text{Planck}}$

Basic Equations

3D equations of radiation MHD in the flux limited diffusion (FLD) approximation

- Eddington tensor: $\mathrm{f} = \frac{1}{2}(1 - f)\mathrm{I} + \frac{1}{2}(3f - 1)nn, \qquad n \equiv \frac{\nabla E}{|\nabla E|}$
- Eddington factor: $f = \lambda(R) + \lambda(R)^2 R^2$

- flux limiter: $\lambda(R) = \frac{2 + R}{6 + 3R + R^2}$

optically **thin** limit $\lim_{R\to\infty} \lambda(R) = \frac{1}{R}, \ \lim_{R\to\infty} f = 1 \quad \Rightarrow \quad |F| = cE$

optically **thick** limit $\lim_{R\to 0} \lambda(R) = \frac{1}{3}, \ \lim_{R\to 0} f = \frac{1}{3} \quad \Rightarrow \quad \mathrm{P} = \frac{1}{3} E\mathrm{I}$

- opacity parameter: $R \equiv \frac{\nabla E}{\chi_{\mathrm{Rosseland}} \rho}$

ZEUS code with FLD module (Turner & Stone 2001) is modified and used.
- energy conservation
- implicit scheme for diffusion equation: Gauss-Seidel method accelerated by FMG

Energy Dissipation

Explicit viscosity and resistivity are not included in the basic equations.
Kinetic and magnetic energies that are numerically lost are captured as internal energy.

$$\begin{cases} \frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2\right) + \nabla \cdot \left(\left(\frac{1}{2}\rho v^2\right)v + pv\right) = (\nabla \cdot v)p - \tilde{Q}_{kin} \\ \frac{\partial}{\partial t}\left(\frac{1}{2}B^2\right) + \nabla \cdot (E \times B) = -\tilde{Q}_{mag} \\ \frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2 + e + \frac{1}{2}B^2\right) + \nabla \cdot \left(\left(\frac{1}{2}\rho v^2 + e\right)v + pv + E \times B\right) = 0 \end{cases}$$

$$\Leftrightarrow \frac{\partial e}{\partial t} + \nabla \cdot (ev) = -(\nabla \cdot v)p + \tilde{Q}_{kin} + \tilde{Q}_{mag}$$

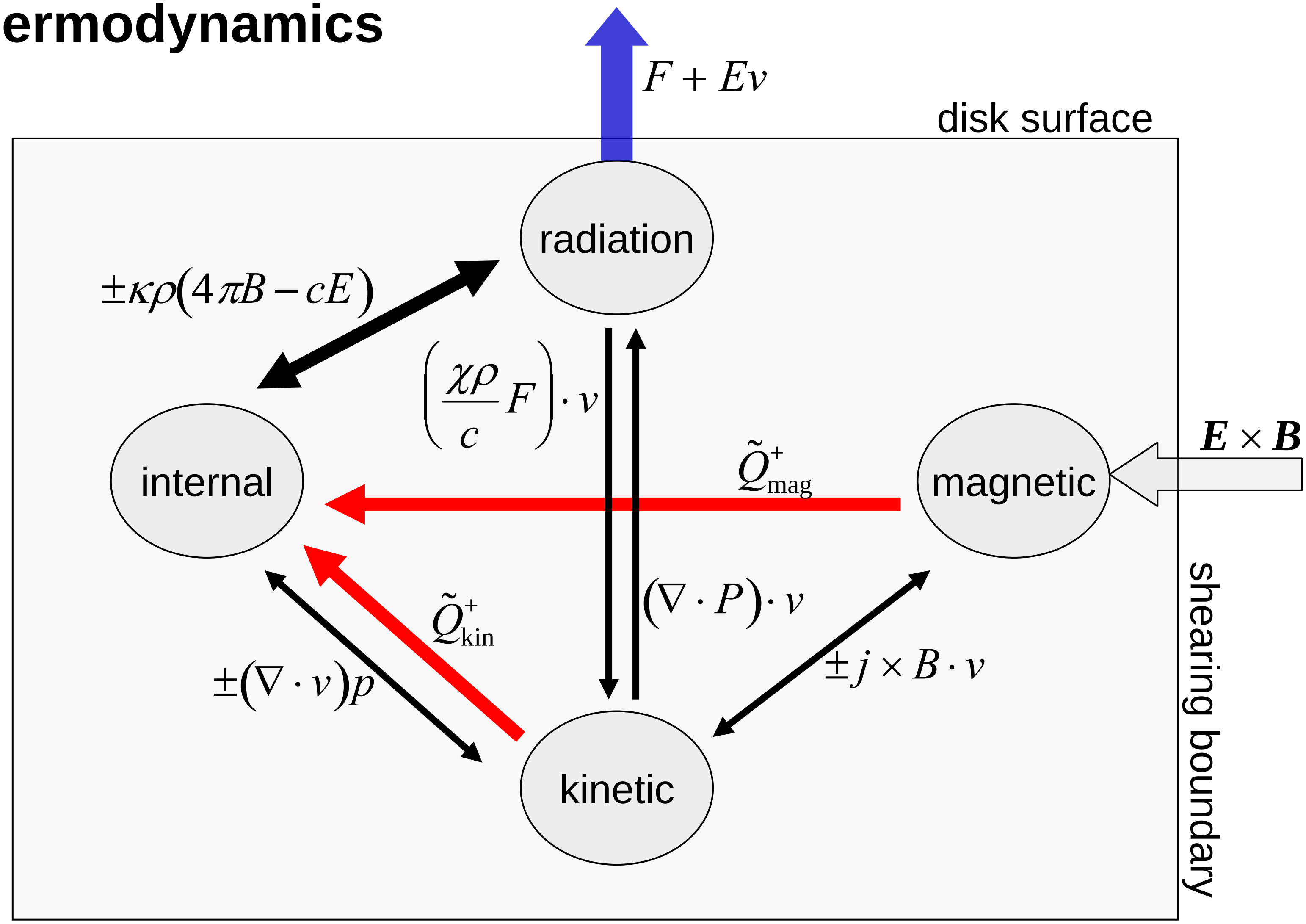
$\tilde{Q}$: numerical dissipation rate

Numerical dissipation rate is evaluated by solving adiabatic equation simultaneously.

$$\frac{\partial e}{\partial t} + \nabla \cdot (e\boldsymbol{v}) = -(\nabla \cdot \boldsymbol{v})p$$

(For clarity, radiation and potential energies are not included in the above.)

Thermodynamics



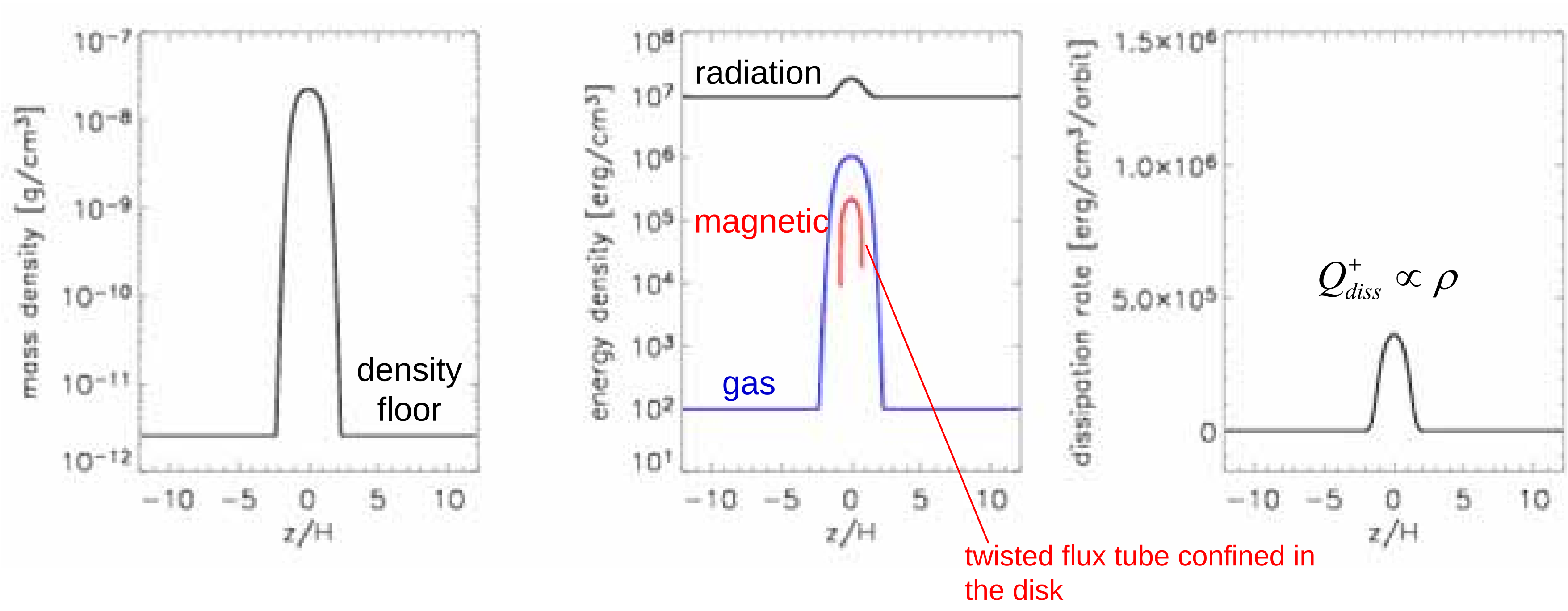
cooling rate $Q^{-}_{\rm rad} = F + Ev$

heating rate $Q^{+}_{\rm diss} = \tilde{Q}_{\rm mag} + \tilde{Q}_{\rm kin}$

Radiation-dominated case

Initial condition (= Shakura – Sunyaev model)

- hydrostatic balance: F_{rad} (+ F_{gas}) + F_{grav} = 0
- energy balance: $Q^{+}_{diss} = Q^{-}_{rad}$
  local dissipation rate $Q_{diss}(z)$ is proportional to density ρ



Radiation-dominated case

Animations on dynamical time scale

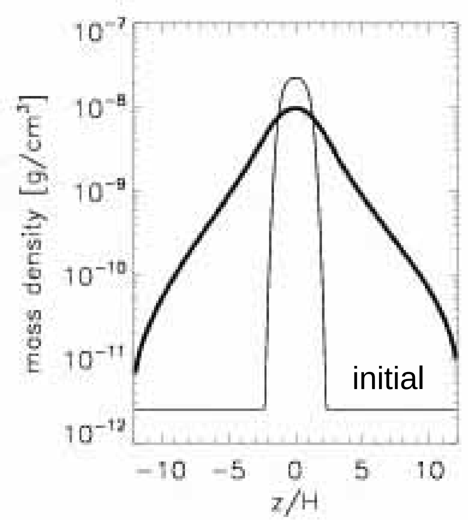
- density
- magnetic energy
- radiation energy
- dissipation rate

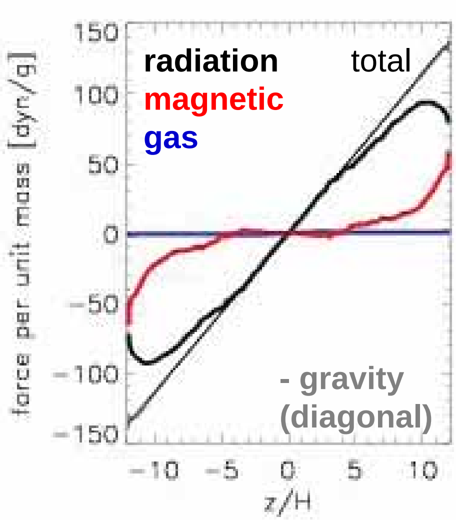
Vertical structure (time-averaged over several thermal time scale)

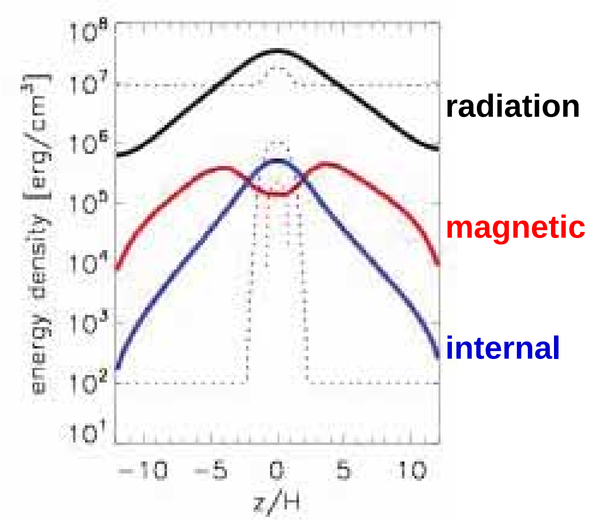
1. **dynamical balance**
2. **energy balance and local dissipation rate**

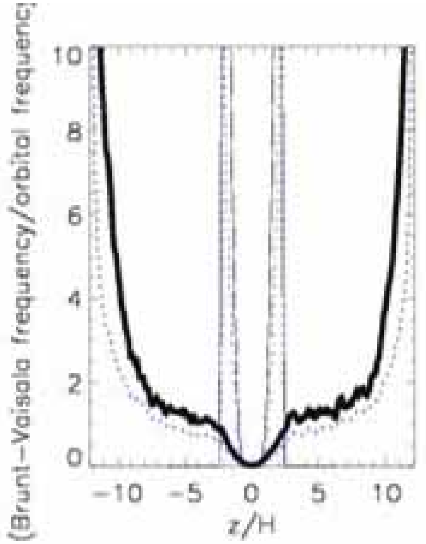
Radiation-dominated case

1. dynamical balance









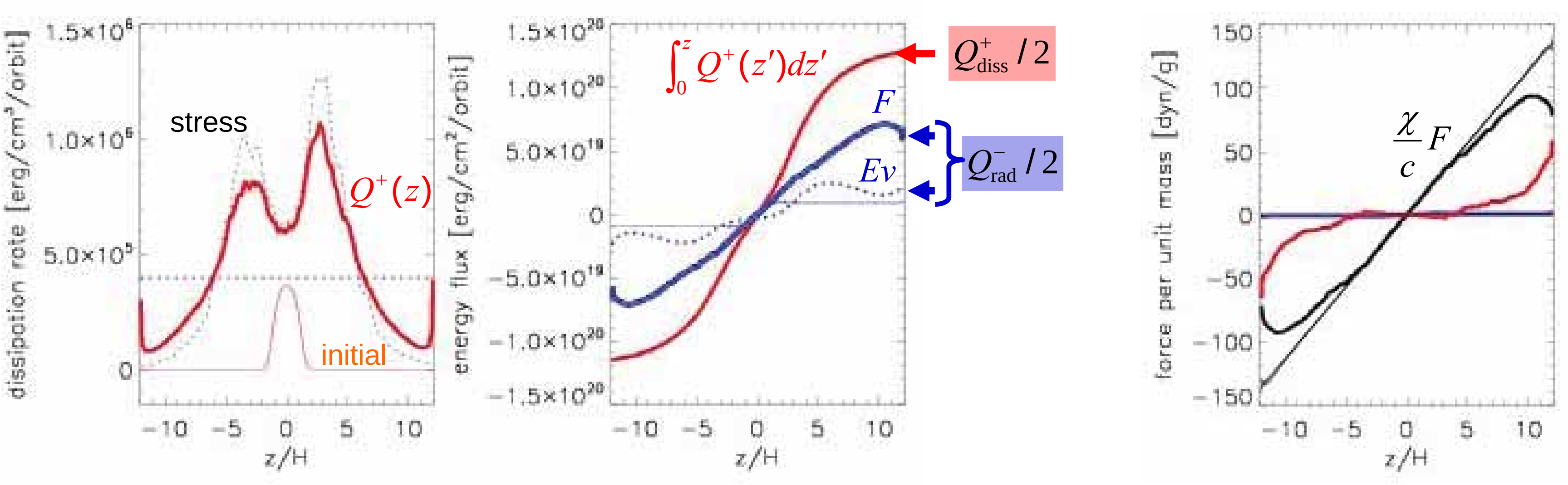
- hydrostatic equilibrium holds

$$\underbrace{\chi F/c}_{|z|<10H} + \underbrace{d/dz(B^2/2)/\rho}_{|z|>10H} \sim \Omega^2 z$$

- stable against convection

Radiation-dominated case

2. energy balance and local dissipation rate



- cooling rate Q^-_{rad} < heating rate Q^+_{diss}

- diffusion flux F (~ radiation force per unit mass) is determined by hydrostatic balance
  $\chi F/c \sim \Omega^2 z$

- **Critical dissipation rate for thermal equilibrium:** $Q^+_{critical}(= dF/dz) \approx c\Omega^2/\chi$
  (Shakura & Sunyaev 1976)

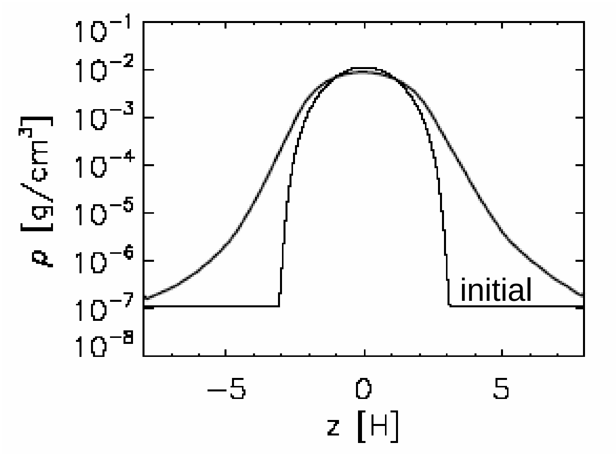
Gas-dominated case

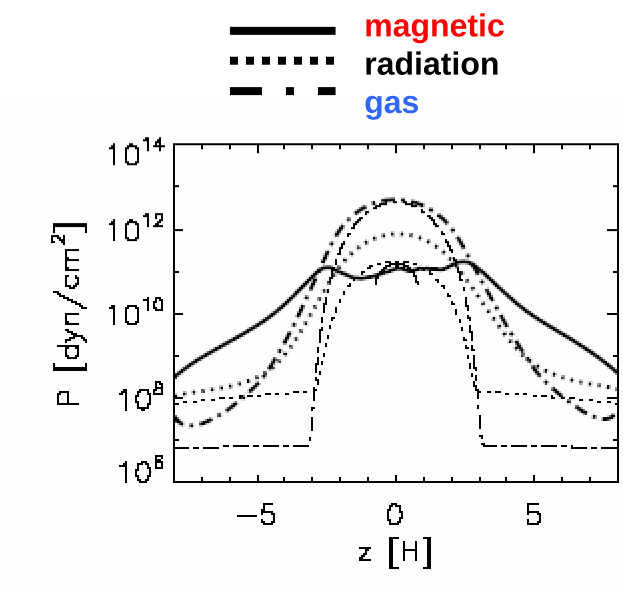
1. dynamical balance

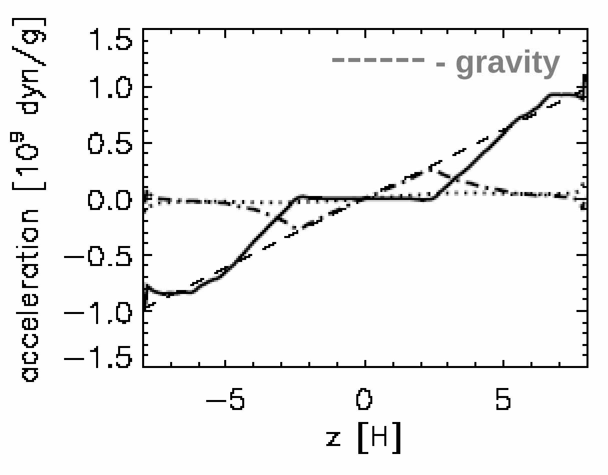
- hydrostatic equilibrium holds

$$\underbrace{dp/dz/\rho}_{|z|<3H} + \underbrace{d/dz(B^2/2)/\rho}_{|z|>3H} \sim \Omega^2 z$$

- stable against convection

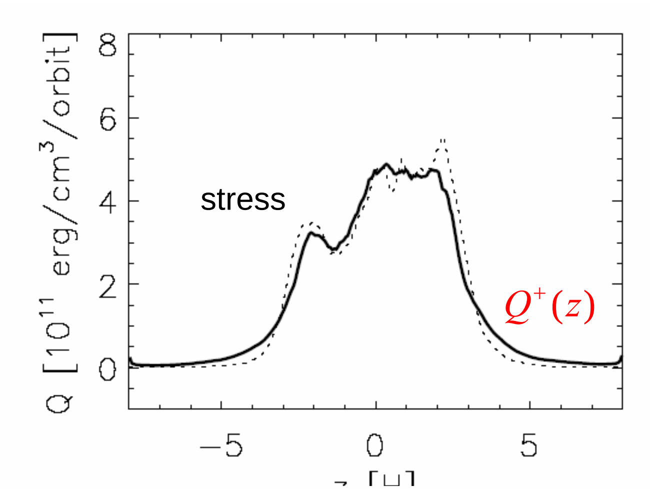


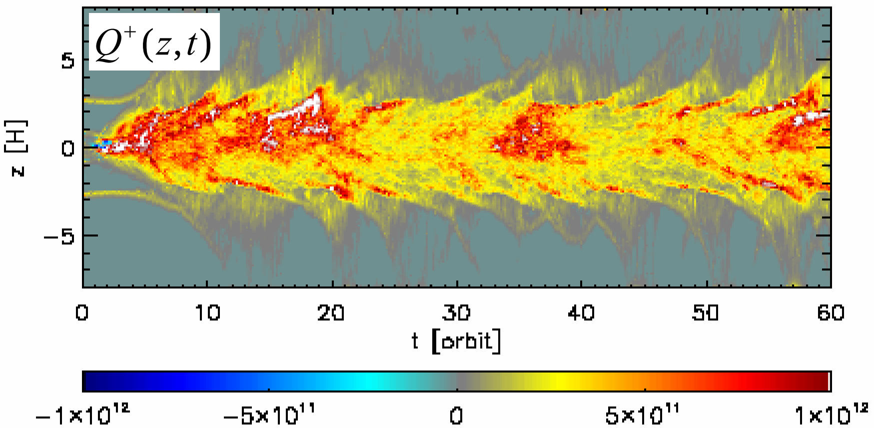


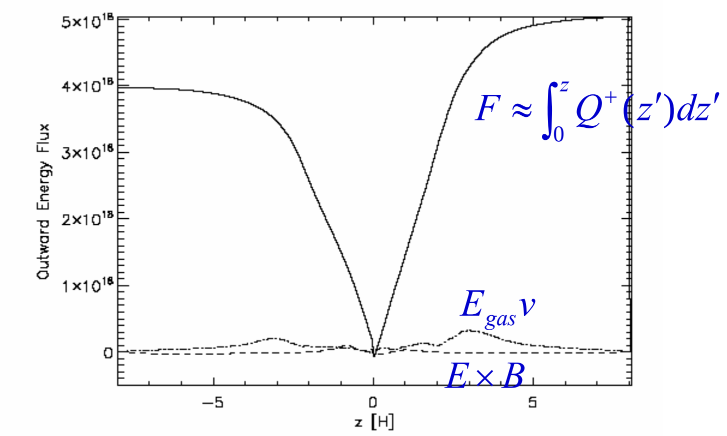


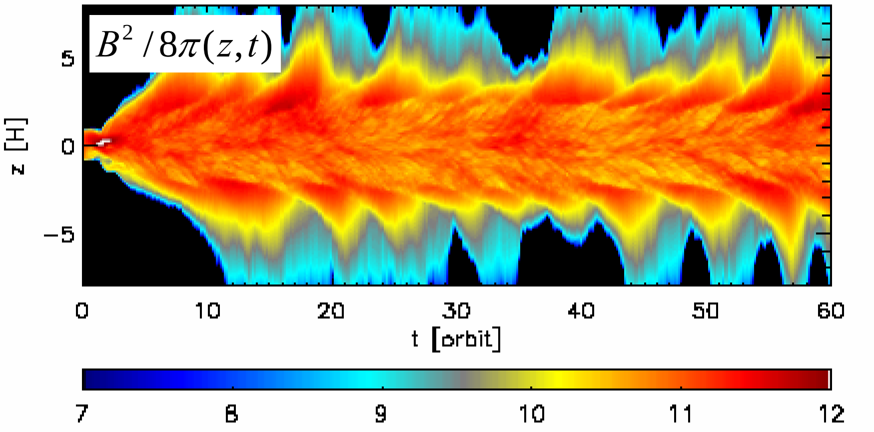
Gas-dominated case

2. energy balance and local dissipation rate



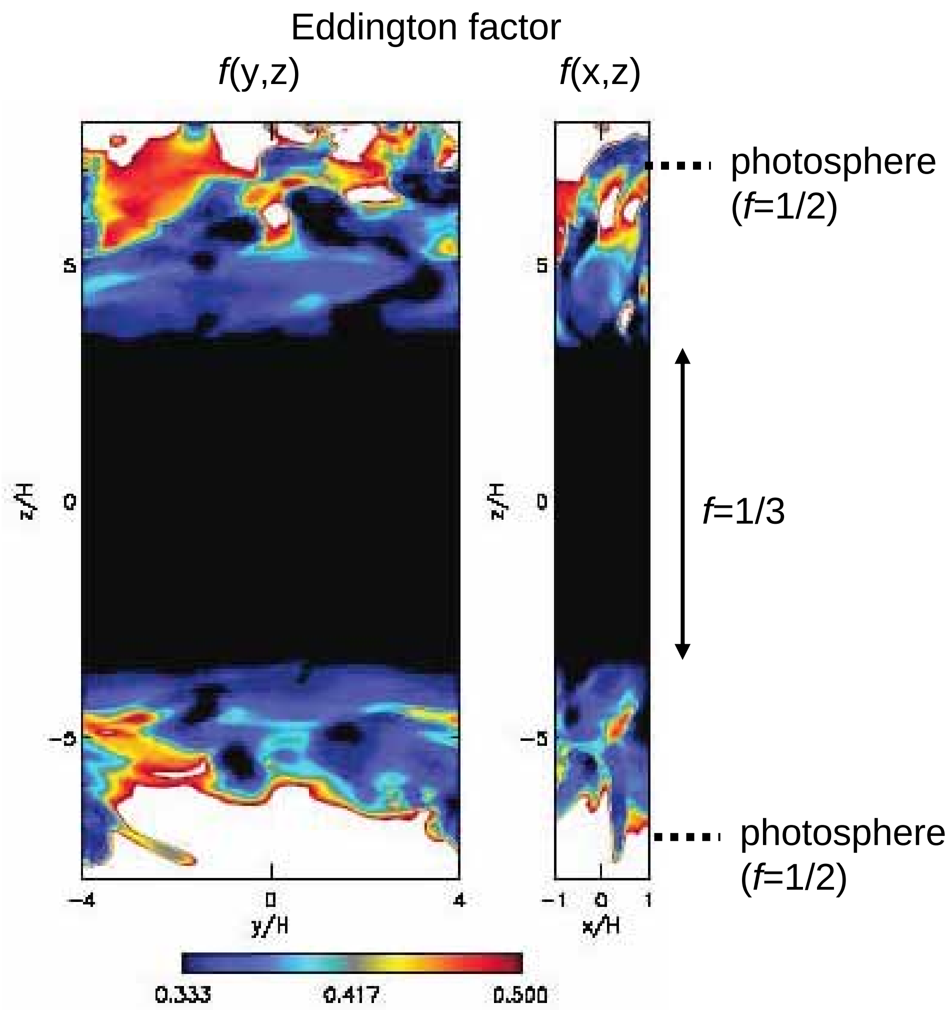




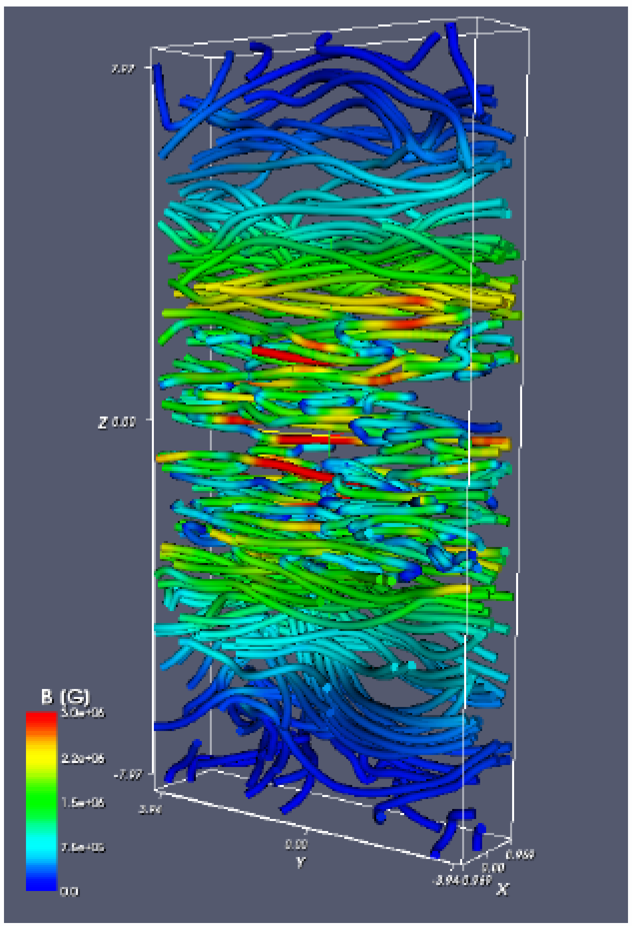


Gas-dominated case

Location of photosphere



Magnetic field structure



Summary

Radiation-dominated disk

<u>Equilibrium state is not obtained</u>.
- hydrostatic balance holds.
- thermal balance *does not* hold.

<u>MRI must generate</u>
<u>the critical dissipation rate</u> $c\Omega^2 / \chi$
<u>for thermal equilibrium??</u>

… Global simulation
is needed to answer the question.

Gas-dominated disk

<u>Equilibrium state is obtained</u>.