カーブラックホール・エルゴ領域に架かる 磁気的橋の爆発的膨張と相対論的ジェット形成

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非常に速く回転するブラックホールのまわりの電流ループの 作る磁場による相対論的ジェット形成:

- 降着円盤を伴うブラックホールのエルゴ領域付近に電流ループがある 場合、エルゴ領域から降着円盤に架かる閉じた磁束管が存在するが、それはエルゴ領域と降着円盤をつなぐ「磁気的橋」と見なす。
- ・磁気的橋はエルゴ領域の空間の引きずり効果により捩じり上げられ爆発的膨張をする。
- 爆発的膨張をしたプラズマは磁気張力により絞られて相対論的ジェット になる。

RMHD mini Workshop @千葉大学(2005.8.9)

#### Motivation: Relativistic Jets in the Universe



Mirabel, Rodriguez 1998

# Relativistic Jets in the Universe

- Active galactic nuclei, Quasars:
  - $\gamma \ge 10$ ,  $L_{iet}$ ~ several M ps
- Microquasars:  $\gamma \sim 3$ ,  $L_{iet} \sim several ps$
- Gamma-ray bursts:  $\gamma \ge 100$ ,  $L_{iet} \sim 1AU$ -several ps

**Relativistic Jet Formation Mechanism** {Acceleration of plasma/gas
{Collimation of plasma/gas outflow

1) Magnetic field

2) Radiation pressure

3) Gas pressure

# Magnetic Formation of Relativistic Jet around Black Hole

- Interaction between plasma and magnetic field around (near) black hole including general relativistic effects
- Most simple approximation: General Relativistic Magnetohydrodynamics (GRMHD)

# Summary of OUR Previous Results with respect to Relativistic Jets

- Sub-relativistic jet (Disk case with Uniform magnetic field) 1999
- 'Poynting flux jet', but No outflow (Uniform plasma with Uniform magnetic field) 2002, 2003
- Relativistic outflow, but No colimated jet (radial magnetic field), 2004
  - No Clear Relativistic Jet

# Realistic Magnetic Configuration of Black Hole Magnetosphere











Hayashi, Shibata, and Matsumoto (1996)

However, black hole does not sustain (dipole) magnetic field.

# Non-relativistic Calculation of Current Loop Case





Kudoh, Matsumoto, & Shibata (2003)





ジェットの形成

ブラックホール近傍の電流ループ が作る磁場配位について

#### 非相対論的電流ループの作る磁場の ベクトル・ポテンシャル

$$A_{\phi} = \frac{\mu_0 I}{\pi} \frac{R_0}{\sqrt{R_0^2 + r^2 + 2R_0 r \sin \theta}} \left[ \frac{(2 - k^2)K(k) - 2E(k)}{k^2} \right]$$



 $s \rightarrow 0 \ cap A_{\phi} \rightarrow \infty$ なので電流ループ付近では 滑らかに有界にする必要がある。

#### 電流ループ近傍で滑らかなベクトル・ポテンシャル

$$A_{\phi}^{\text{nonrela, current loop}} = \frac{\mu_0 I}{\pi} \frac{R_0}{\sqrt{R_0^2 + r^2 + 2R_0 r \sin \theta}} \begin{bmatrix} (2 - k^2)K(k) - 2E(k) \\ k^2 \end{bmatrix} \Phi(k)$$

$$k^2 = \frac{4R_0 r \sin \theta}{R_0^2 + r^2 + 2R_0 r \sin \theta}$$

$$s \to 0 \ (r \to R_0, \ \theta \to \pi/2)$$

$$k \to 1$$

$$\Phi \to \infty$$

$$A_{\phi} \to \infty$$

$$A_{\phi} = \frac{\mu_0 I}{\pi} \frac{R_0}{\sqrt{R_0^2 + r^2 + 2R_0 r \sin \theta}} \Phi(\tilde{k})$$

### ブラックホールのまわりの電流ループが 作る磁場のベクトルポテンシャル

- カー時空において次の条件を満たすBを与えたい:
  - ∇·**B**=0を満たす<sup>Ѻ</sup>ベクトルポテンシャル, B=∇×A
  - 電流ループの外ではcurrent-freeに近い,  $\nabla \times (\alpha B) = 0$
  - 電流ループの作る磁場のベクトルポテンシャルは次のよう な性質を持つ。

ブラック

 $( \bullet )$ 

 $\langle \! \otimes \! \rangle$ 

$$A_{\phi} \to A_{\phi}^{\text{nonrela, current loop}} \qquad (r >> r_{\text{H}})$$

$$A_{\phi} \to A_{\phi}^{\text{uniform}} \qquad (r \to r_{\text{H}})$$

 $A_{\phi}^{uniform}$ はブラックホールの十分遠方で磁場の強さが  $B_0$ の一様磁場を与えるベクトルポテンシャルの方位角成分:

$$A_{\phi}^{\text{uniform}} = \frac{B_0}{2} h_{\phi} \left( 1 - \frac{2ar_g \alpha \beta^{\phi}}{h_{\phi}} \right) = \frac{B_0}{2} h_{\phi} \left( 1 - \frac{4r_g (ar_g)^2 r}{A} \right)$$

なので,  $A_{\phi} = 2 \frac{A_{\phi}^{\text{uniform}} / B_{0}}{h_{\phi}^{\text{nonrela}}} A_{\phi}^{\text{nonrela, current loop}}$   $= \frac{h_{\phi}}{h_{\phi}^{\text{nonrela}}} \left(1 - \frac{4r_{g}(ar_{g})^{2}r}{A}\right) A_{\phi}^{\text{nonrela, current loop}}$ 

とすればよい。

### Magnetic Field induced by Current Loop around Kerr Black Hole



#### **Current Density of Initial Magnetic Field**



ブラックホールのまわりの プラズマの平衡解・準平衡解

#### Quasi-hydrostatic Equilibrium of Plasma around Kerr Black Hole

Hydrostatic Equilibrium of Plasma

$$\rho = \rho_0 \left( \alpha^{-\frac{\Gamma}{(\Gamma_0 + 1)(\Gamma - 1)}} - 1 \right)^{\Gamma_0}$$
$$p = \frac{\Gamma - 1}{\Gamma} \rho_0 c^2 \left( \alpha^{-\frac{\Gamma}{(\Gamma_0 + 1)(\Gamma - 1)}} - 1 \right)^{\Gamma_0 + 1}$$

- $\alpha$  : lapse function
- $\Gamma$ : Specific heat ratio, 5/3
- $\Gamma_0$ : Positive constant

But, these variables are infinite at horizon. Modification to finite variables at horizon

→ Quasi-hydrostatic equilibrium

$$\begin{aligned} \alpha \to \alpha(\tilde{r}, \theta) \\ \tilde{r} &= r + (r_{\rm smt} - r_{\rm H}) \exp(-(r - r_{\rm H})/(r_{\rm smt} - r_{\rm H})) \\ \tilde{r} \to r_{\rm smt} \quad (r \to r_{\rm H}) \end{aligned}$$

# GRMHD方程式



#### Base of General Relativistic MHD in Kerr Space-Time

• General relativistic equation of conservation laws and Maxwell equations:

 $\nabla_{\nu}(n U^{\nu}) = 0 \qquad \text{(conservation of particle number)}$   $\nabla_{\nu} T^{\mu\nu} = 0 \qquad \text{(conservation of energy and momentum)}$   $\partial_{\mu} F_{\nu\lambda} + \partial_{\nu} F_{\lambda\mu} + \partial_{\lambda} F_{\mu\nu} = 0 \qquad \text{(Maxwell equations)}$  $\nabla_{\mu} F^{\mu\nu} = -J^{\nu}$ 

- Frozen-in condition:  $F_{vu}U^v = 0$
- Kerr Metric:  $ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu}$ ;  $g_{\mu\nu} = -h_0^2$ ;  $g_{\mu\nu} = -h_1^2$ :

$$g_{00} = -h_0$$
;  $g_{ii} = -h_i$ ;  
 $g_{0i} = -h_i^2 \omega_i$  (i=1,2,3);  $g_{ij} = 0$  (i $\neq j$ )

- *n*: proper particle number density. p: proper pressure. c: speed of light.
- *e* : proper total energy density,  $e=mnc^2 + p / (\Gamma 1)$ .
- *m* : rest mass of particles.  $\Gamma$ : specific heat ratio.
- $U^{\mu\nu}$ : velocity four vector.  $A^{\mu\nu}$ : potential four vector.  $J^{\mu\nu}$ : current density four vector.
- $\nabla^{\mu\nu}$ : covariant derivative.  $g_{\mu\nu}$ : metric.
- $T^{\mu\nu}$ : energy momentum tensor,  $T^{\mu\nu} = pg^{\mu\nu} + (e+p)U^{\mu}U^{\nu} + F^{\mu\sigma}F^{\nu}{}_{\sigma} g_{\mu\nu}F^{\lambda\kappa}F_{\lambda\kappa}$ }/4.
- $F_{\mu\nu}$ : field-strength tensor,  $F_{\mu\nu} = \partial_{\mu} A_{\nu} \partial_{\nu} A_{\mu}$ .

Vector Form of General Relativistic MHD Equation  
(3+1 Formalism) ~ similar to Classical MHD  

$$\frac{\partial D}{\partial t} = -\nabla \cdot [\underline{\alpha} D(\mathbf{v} + \mathbf{v}_{H})]$$
 (conservation of particle number)  
 $\frac{\partial \mathbf{P}}{\partial t} = -\nabla \cdot [\underline{\alpha} (\mathbf{T} + \underline{c\beta} \mathbf{P})] - (D + \frac{\varepsilon}{c^{2}}) \nabla (c^{2} \alpha) + \underline{\alpha} \mathbf{f}_{curv} - \mathbf{P} : \mathbf{\sigma}$  (equation of motion)  
 $\frac{\partial \mathbf{P}}{\partial t} = -\nabla \cdot [\underline{\alpha} (\mathbf{T} + \underline{c\beta} \mathbf{P})] - (D + \frac{\varepsilon}{c^{2}}) \nabla (c^{2} \alpha) + \underline{\alpha} \mathbf{f}_{curv} - \mathbf{P} : \mathbf{\sigma}$  (equation of motion)  
Special relativistic total energy density special relativistic effect  
 $\frac{\partial \varepsilon}{\partial t} = -\nabla \cdot [\underline{\alpha} (c^{2} \mathbf{P} - Dc^{2} \mathbf{v} + \underline{e} \mathbf{v}_{H})] - (\nabla \alpha) \cdot c^{2} \mathbf{P} - \mathbf{T} : \mathbf{\sigma}$  (equation of energy)  
 $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [\underline{\alpha} (\mathbf{c} - \underline{c} \mathbf{\beta} \times \mathbf{B})]$   $\mathbf{J} + \underline{\rho_{e} c} \mathbf{\beta} + \frac{1}{c^{2}} \frac{\partial \mathbf{E}}{\partial t} = \nabla \times \left[ \underline{\alpha} \left( \mathbf{B} + \frac{\mathbf{\beta}}{c} \times \mathbf{E} \right) \right]$   
 $\nabla \cdot \mathbf{B} = \mathbf{0}$   $\rho_{e} = \frac{\underline{\alpha}}{c^{2}} \nabla \cdot \mathbf{E}$  (Maxwell equations)  
 $\mathbf{E} + \mathbf{v} \times \mathbf{B} = \mathbf{0}$  (ideal MHD condition)

where

-

$$\alpha = \sqrt{h_0^2 + \sum_{i=1}^3 \left(\frac{h_i \omega_i}{c}\right)^2} : \text{(Lapse function)} \qquad \beta^i = \frac{h_i \omega_i}{c \alpha} : \text{(shift vector)} \qquad \mathbf{v}_{\mathrm{H}} = c \boldsymbol{\beta} : \text{(shift velocity)}$$
$$f_{\mathrm{curv}}^i = \sum_{j=1}^3 \left(G_{ij} T^{ij} - G_{ji} T^{jj}\right) \qquad G_{ij} = -\frac{1}{h_i h_j} \frac{\partial h_i}{\partial x^j} \qquad \sigma_{ij} = \frac{h_i}{h_j} \frac{\partial \omega_i}{\partial x^j}$$

# 準静水圧平衡コロナでの計算 テスト計算→数値計算

# **Initial Condition**

- Black Hole:  $a \equiv \frac{J}{J_{\text{max}}} = 0.99995$  (Almost maximally rotating)
- Magnetic Field: Magnetic field induced by current loop around black hole ( $J_0=1.5\pi/2$ ,  $R_0=r_S$ ,  $\delta=0.5r_S$ )
- Plasma:
  - Corona

quasi-hydrostatic equilibrium ( $\Gamma_0$ =5,  $\rho_0$ =0.018,  $r_{smt}$ =0.8 $r_s$ )  $\hat{\mathbf{v}} = 0$ 

– Disk

 $\rho_{\text{disk}} = 100 \rho_{\text{corona}}, p_{\text{disk}} = p_{\text{corona}}$  $\hat{\mathbf{v}}_{\text{P}} = 0, \qquad v_{\phi} = \pm v^{\pm}_{\text{Kepler}} \begin{cases} \text{Co-rotating disk} \\ -\text{Counter-rotating disk} \end{cases}$ 

# **Boundary Condition**

• Calculation region:

 $1.016r_{\rm H} \le r \le 80r_{\rm H}$ 

 $0.01 \le \theta \le \pi \, / \, 2$ 

• Boundary condition:

Radial boundary condition: free boundary condition

Axi-symmetric and mirror boundary conditions at  $\theta$  =0.01,  $\pi/2$ 

# 完全静水圧平衡のテスト *t* =0 *t* =30<sub>てs</sub>



Color:  $\log \rho$ ,



Arrows: Velocity

# 準静水圧平衡のテスト

*t* =0

 $t = 24.23\tau_{\rm S}$ 





Color:  $\log \rho$ , Arrows: Velocity

# 順方向回転円盤

*t* =0

 $t = 15.49 \tau_{\rm S}$ 





Color:  $\log \rho$ , Arrows: Velocity





### **Current Density of Initial Magnetic Field**







Solid line: Magnetic field line Color:  $\log \rho$ , Arrows: Velocity

 $v^{max} = 0.580c$  $v_{P}^{max} = 0.370c$ 



Solid line: Magnetic Field line Color: log  $\rho$ Arrows: Velocity

# Azimuthal component of Magnetic Field t=0 $t=10.33\tau_{S}$





Color:  $B_{\phi}^{2}/\rho c^{2}$ Solid line: Magnetic field line



Solid line: Magnetic field line Color:  $B_{\phi}^{2}/\rho c^{2}$ Arrows: Velocity

1.0 2.0 3.0 4.0 5.0



What happen after the final time of present calculations?

# Non-relativistic Calculation of Current Loop Case





Kudoh, Matsumoto, & Shibata (2003)





ジェットの形成

# 静水圧平衡コロナでの計算 計算結果 (長時間の追跡が可能)

# **Initial Condition**

- Black Hole:  $a \equiv \frac{J}{J_{\text{max}}} = 0.99995$  (Almost maximally rotating)
- Magnetic Field: Magnetic field induced by current loop around black hole ( $J_0=1.5\pi/2$ ,  $R_0=r_S$ ,  $\delta=0.5r_S$ )
- Plasma:
  - Corona

hydrostatic equilibrium (  $\Gamma_0=\!4,\,\rho_0\!=\!0.018)$   $\hat{\mathbf{v}}=0$ 

– Disk

 $\rho_{\text{disk}} = 300 \rho_{\text{corona}}, p_{\text{disk}} = p_{\text{corona}}$  $\hat{\mathbf{v}}_{p} = 0, \qquad v_{\phi} = v^{\pm}_{\text{Kepler}} \begin{cases} \text{Co-rotating disk} \\ \hline \text{Counter-rotating disk} \end{cases}$ 





Solid line: Magnetic field line

Color: log  $\rho$ 

*t* =0

# 磁気配位,速度,質量密度分布









5 10 15 20

/r R





# 磁気配位,磁場の方位角成分



Solid line: Magnetic field line

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Color: B_{\phi}/\rho^{1/2}
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t = 0

# 磁気配位, 磁場の方位角成分 *t* =57.6<sub>τs</sub>











#### ジェット 絞込みの機構の同定: ジェット 収束の曲率(非相対論)







(n·v=0)



#### 

# 磁気配位, 磁場の方位角成分 *t* =57.6<sub>τs</sub>



なぜ磁気的橋は内側の端で主にねじられるのか?#1

単位時間単位面積当たりの円盤から磁束管へ注入されるエネルギーの流入量は



なぜ磁気的橋は内側の端で主にねじられるのか?#2



磁力線の外側の端で 回転しようと、内側の 橋で回転しようと同じ 磁場配位になる。



内側を回転させるほうが単位時 間当たり回転数が大きく出来るの で内側の端を回転させるほうが 磁場を効率よく捩じることになる。

すなわち、磁気的橋はエルゴ領域で 捩じられることになる。

パラメータ依存性

#### 静水圧平衡コロナ・順方向回転





#### 円盤の質量密度の依存性









 $t = 47\tau_{\rm S}$ 





#### コロナの圧力・密度分布の依存性





# **Summary and Speculation**

- Sub-relativistic jets were formed from the magnetic bridge between the ergosphere and disk around rapidly rotating black hole.
- The magnetic bridges between the ergosphere and the disk are twisted by the frame-dragging effect in the ergosphere. The twist by the disk is negligible in the present cases.
- The magnetic pressure of the twisted magnetic flux tubes brows off the plasma around the black hole.
- The outflow is collimated by the magnetic force to become (sub-relativistic) jet.

Speculation for further work toward relativistic jet: •Stronger magnetic field  $\rightarrow$  Relativistic acceleration (parameter scan with respect to  $J_0$ )



# ブラックホール磁気圏: 宇宙で最もダイナミックで活発な領域!



Expected phenomena caused by closed magnetic field of current loop near rotating black hole





Magnetic reconnection should be important near black hole horizon!

#### • Magnetic reconnection

- Electric resistivity
- → Resistive relativistic MHD:



