# Local Dissipation and Radiative Transfer in Accretion Disks

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# Radial Structure of Geometrically Thin and Optically Thick Disks

One zone model by Shakura and Sunyaev (1973)

- local energy balance: $Q^{+}_{\text{vis}}(r) = Q^{-}_{\text{rad}}(r)$
- hydrostatic balance: $\Omega_{\text{K}}(r)\, H(r) = c_{\text{s}}(r)$
- "α" viscosity: $T_{r\phi}(r) = -\alpha\, p(r)$

$$
\begin{cases}
H(r) = H(r; M, \dot{M}, \alpha) \\
\Sigma(r) = \Sigma(r; M, \dot{M}, \alpha) \\
\cdots
\end{cases}
$$

(a) inner region: $p \approx p_{\text{radiation}},\ \chi \approx \chi_{\text{Thomson\_scattering}}$
(b) middle region: $p \approx p_{\text{gas}},\ \chi \approx \chi_{\text{Thomson\_scattering}}$
(c) outer region: $p \approx p_{\text{gas}},\ \chi \approx \chi_{\text{free\_free}}$

# Vertical Stratification of the Disks

$$\begin{cases} -\dfrac{dp(z)}{dz} + \dfrac{\chi(z)\rho(z)}{c} F(z) - \rho(z)\Omega_{\mathrm{K}}^2 z = 0 & \text{momentum equation for gas} \\ -\left(4\pi B(z) - cE(z)\right)\kappa(z)\rho(z) + Q_{\mathrm{diss}}^{+}(z) = 0 & \text{energy equation for gas} \\ -\dfrac{dP(z)}{dz} - \dfrac{\chi(z)}{c} F(z) = 0 & \text{momentum equation for radiation} \\ \left(4\pi B(z) - cE(z)\right)\kappa(z)\rho(z) - \dfrac{dF(z)}{dz} = 0 & \text{energy equation for radiation} \end{cases}$$

**MHD turbulence driven by MRI:**
**most promising candidate for viscosity**

Dynamical equations of gas, radiation and magnetic field must be solved self-consistently.

## Purpose of This Work is to Obtain …

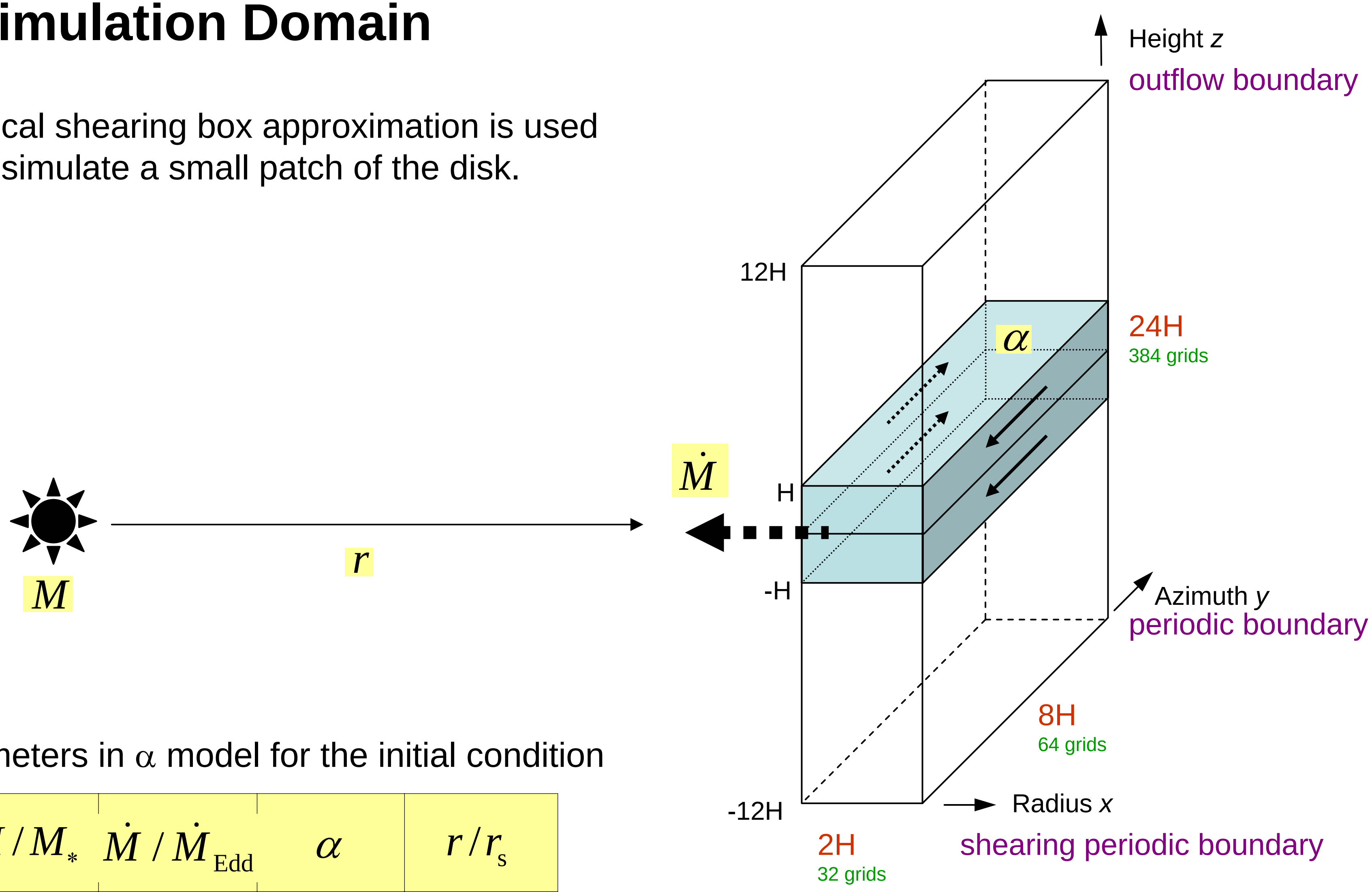
Vertical stratification of MRI-driven (gas-dominated) disks
using 3D Radiation MHD simulation with FLD approximation, where
dissipation process and radiative transfer are explicitly solved.
( Hirose, Krolik and Stone 2006 )

Related works

- Miller & Stone (2000): gas-dominated disk (iso-thermal)
- Turner (2004): radiation-dominated disk (FLD)

# Simulation Domain

Local shearing box approximation is used
to simulate a small patch of the disk.




Parameters in $\alpha$ model for the initial condition

| $M / M_*$ | $\dot{M} / \dot{M}_{\mathrm{Edd}}$ | $\alpha$ | $r / r_s$ |
|---|---|---|---|
| 6.62 | 0.1 | 0.03 | 300 |

# Basic Equations

Radiation MHD in the frequency-averaged flux limited diffusion (FLD) approximation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$\frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho v v) = -\nabla p + j \times B + \frac{\bar{\chi}_{\mathrm{Rosseland}} \rho}{c} F - 2\rho \Omega \times v + 3\rho \Omega^2 x - \rho \Omega^2 z$$

$$\frac{\partial E}{\partial t} + \nabla \cdot (E v) = -\nabla v : \mathrm{P} + \bar{\kappa}_{\mathrm{Planck}} \rho (4\pi B_{\mathrm{Planck}} - cE) - \nabla \cdot F$$

$$\frac{\partial e}{\partial t} + \nabla \cdot (e v) = -(\nabla \cdot v) p - \bar{\kappa}_{\mathrm{Planck}} \rho (4\pi B_{\mathrm{Planck}} - cE)$$

$$\frac{\partial B}{\partial t} - \nabla \times (v \times B) = 0$$

$$F = -\frac{c\lambda}{\bar{\chi}_{\mathrm{Rosseland}} \rho} \nabla E$$

$$p = (\gamma - 1) e$$

$$\mathrm{P} = \mathrm{f}\ E$$

- LTE: source function = Planck Function $B_{\mathrm{Planck}}$
- energy-mean opacity = Planck-mean opacity: $\bar{\kappa}_E = \bar{\kappa}_{\mathrm{Planck}}$

# Basic Equations

3D equations of radiation MHD in the flux limited diffusion (FLD) approximation

- Eddington tensor: $\mathrm{f} = \frac{1}{2}(1 - f)\mathrm{I} + \frac{1}{2}(3f - 1)nn,\qquad n \equiv \frac{\nabla E}{|\nabla E|}$
- Eddington factor: $f = \lambda(R) + \lambda(R)^2 R^2$

- flux limiter: $\lambda(R) = \dfrac{2+R}{6+3R+R^2}$

optically **thin** limit $\lim_{R\to\infty}\lambda(R) = \frac{1}{R},\ \lim_{R\to\infty} f = 1 \quad\Rightarrow\quad |F| = cE$

optically **thick** limit $\lim_{R\to 0}\lambda(R) = \frac{1}{3},\ \lim_{R\to 0} f = \frac{1}{3} \quad\Rightarrow\quad \mathrm{P} = \frac{1}{3}E\mathrm{I}$

- opacity parameter: $R \equiv \dfrac{\nabla E}{\chi_{\mathrm{Rosseland}}\rho}$

ZEUS code with FLD module (Turner & Stone 2001) is modified and used.
- energy conservation
- implicit scheme for diffusion equation: Gauss-Seidel method accelerated by FMG

# Energy Dissipation

Explicit viscosity and resistivity are not included in the basic equations.
Kinetic and magnetic energies that are numerically lost are captured as internal energy.

$$\left\{\begin{aligned}
&\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2\right)+\nabla\cdot\left(\left(\frac{1}{2}\rho v^2\right)v+pv\right)=(\nabla\cdot v)p-\tilde{Q}_{kin}\\
&\frac{\partial}{\partial t}\left(\frac{1}{2}B^2\right)+\nabla\cdot(E\times B)=-\tilde{Q}_{mag}\\
&\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+e+\frac{1}{2}B^2\right)+\nabla\cdot\left(\left(\frac{1}{2}\rho v^2+e\right)v+pv+E\times B\right)=0
\end{aligned}\right.$$

$$\Leftrightarrow \frac{\partial e}{\partial t}+\nabla\cdot(ev)=-(\nabla\cdot v)p+\tilde{Q}_{kin}+\tilde{Q}_{mag}$$

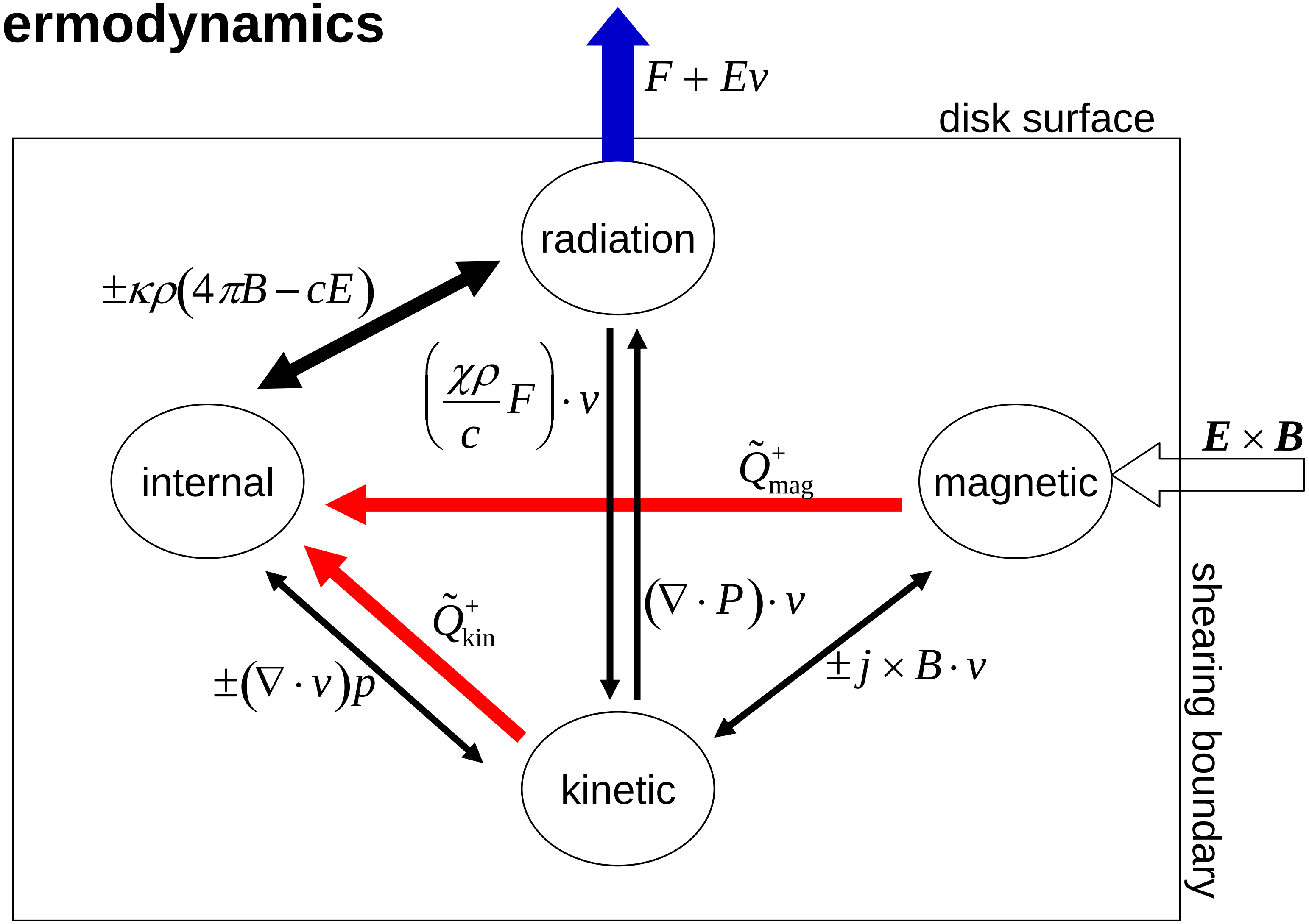
$\tilde{Q}$ : numerical dissipation rate

Numerical dissipation rate is evaluated by solving adiabatic equation simultaneously.

$$\frac{\partial e}{\partial t}+\nabla\cdot(e\boldsymbol{v})=-(\nabla\cdot\boldsymbol{v})p$$

(For clarity, radiation and potential energies are not included in the above.)

# Thermodynamics



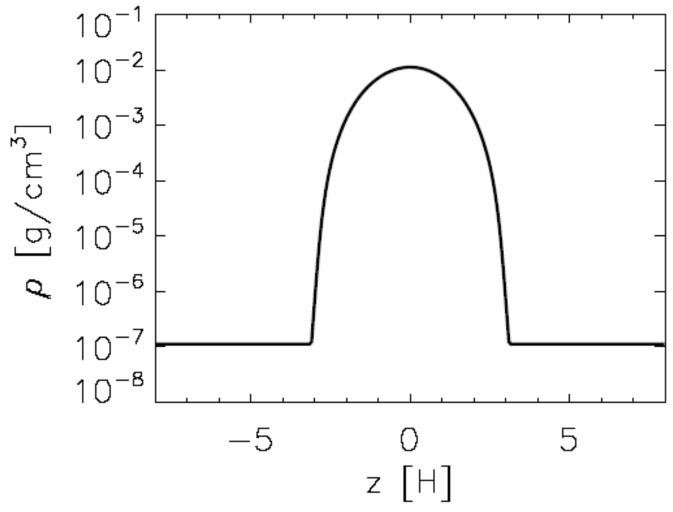

**cooling rate** $Q^{-}_{\rm rad} = F + Ev$

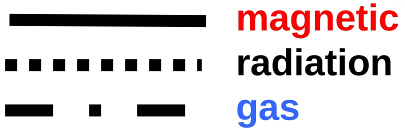
**heating rate** $Q^{+}_{\rm diss} = \tilde{Q}_{\rm mag} + \tilde{Q}_{\rm kin}$

# Initial Condition

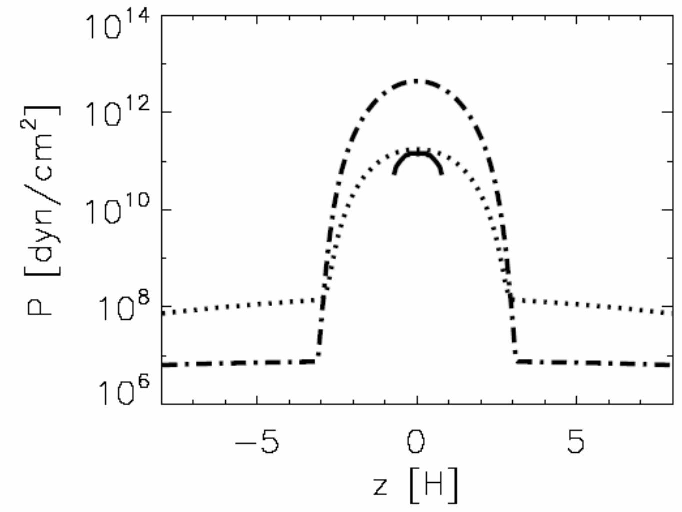
$$\begin{cases} -\dfrac{dp}{dz} + \dfrac{\chi\rho}{c} F - \rho \Omega_{\mathrm{K}}^2 z = 0 & \text{hydrostatic balance} \\ -\dfrac{dF}{dz} + Q_{\mathrm{diss}}^{+}(z) = 0 & \text{energy balance} \\ F = -\dfrac{c}{3\chi\rho}\dfrac{dE}{dz} & \text{Eddington approximation} \\ E = aT^4 & T_{\mathrm{gas}} = T_{\mathrm{rad}} \end{cases}$$

**density**

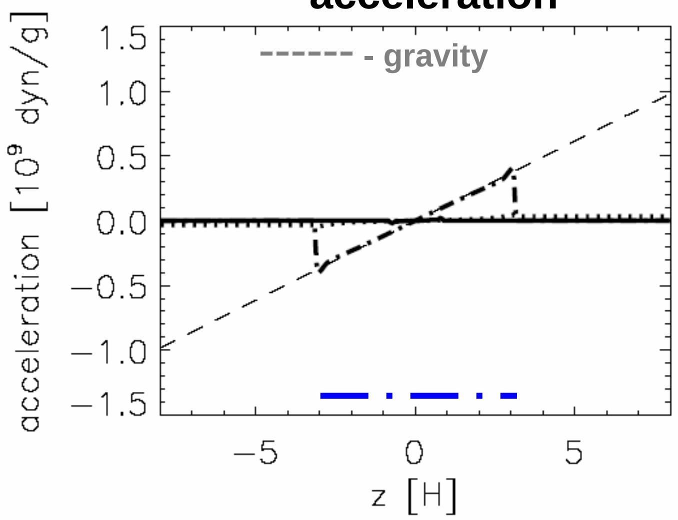



**pressure**




**acceleration**



## Time Evolution of the System

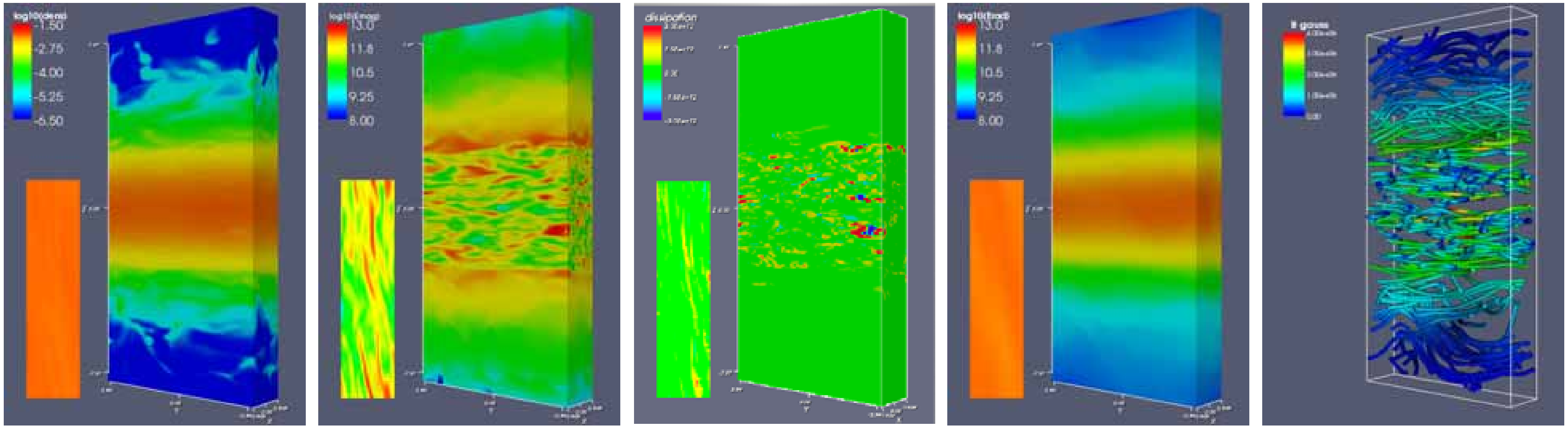
T=0 10 60 orbits

Initial transient

statistically steady state
( ~ 5 cooling time )

## MHD Turbulence driven by MRI (T=25-30 orbits)

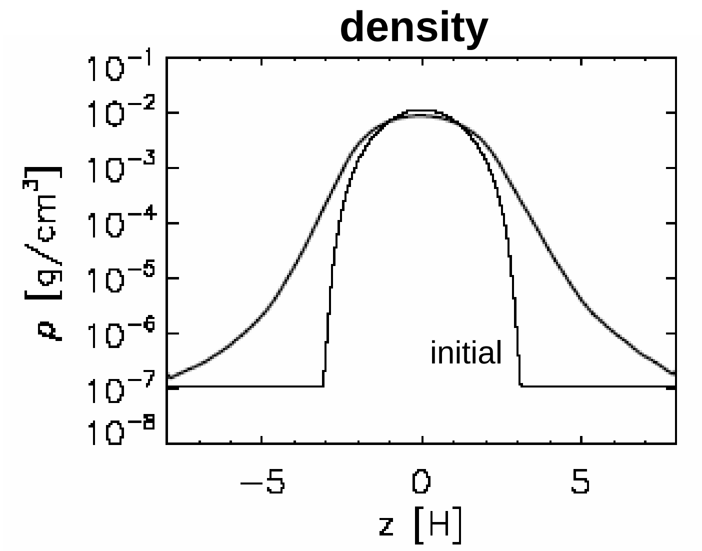
density magnetic energy dissipation rate radiation energy magnetic field lines

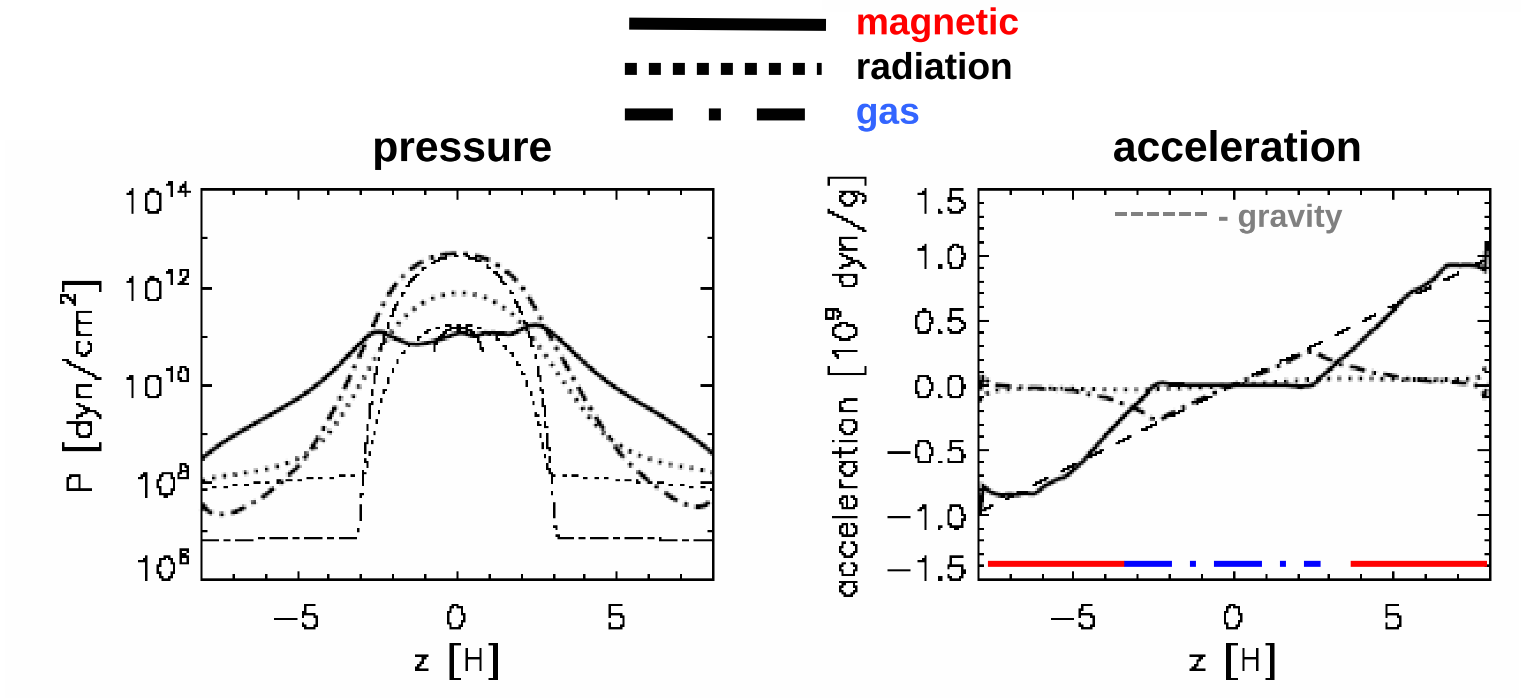


# Hydrostatic Balance

- |z| < 3H (disk body)
  - $\beta > 1$
  - gas pressure grad. ~ gravity
  - constant magnetic pressure

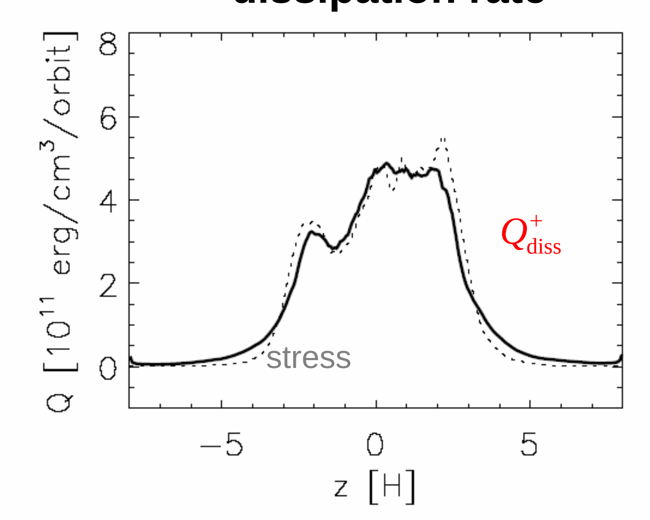
- |z| > 3H (disk atmosphere)
  - $\beta < 1$
  - magnetic pressure grad. ~ gravity



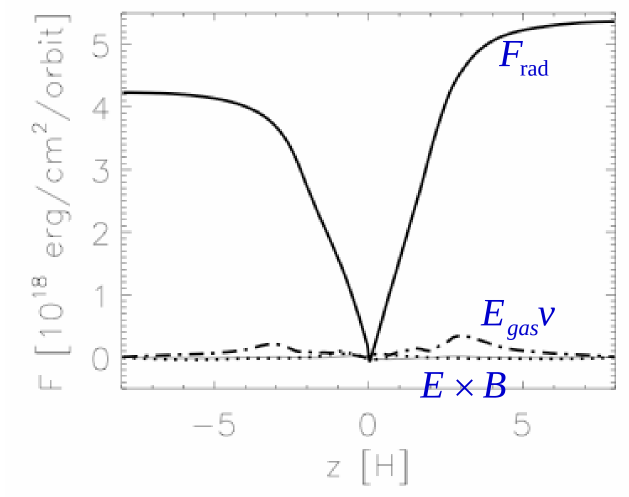
# Local Dissipation and Energy Balance

**dissipation rate**



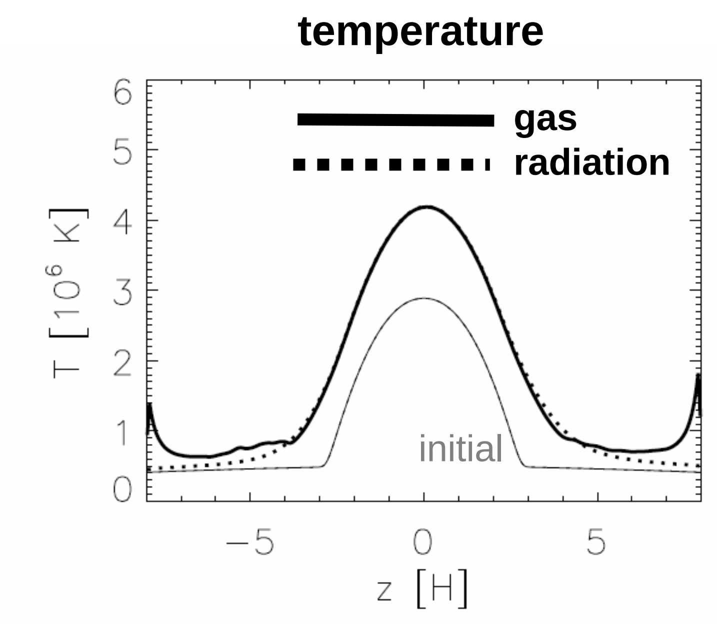

- Dissipation occurs mainly inside the disk body.
- Dissipation rate is roughly uniform inside the disk body.
- Dissipation distribution well agrees with stress distribution.

**energy flux**



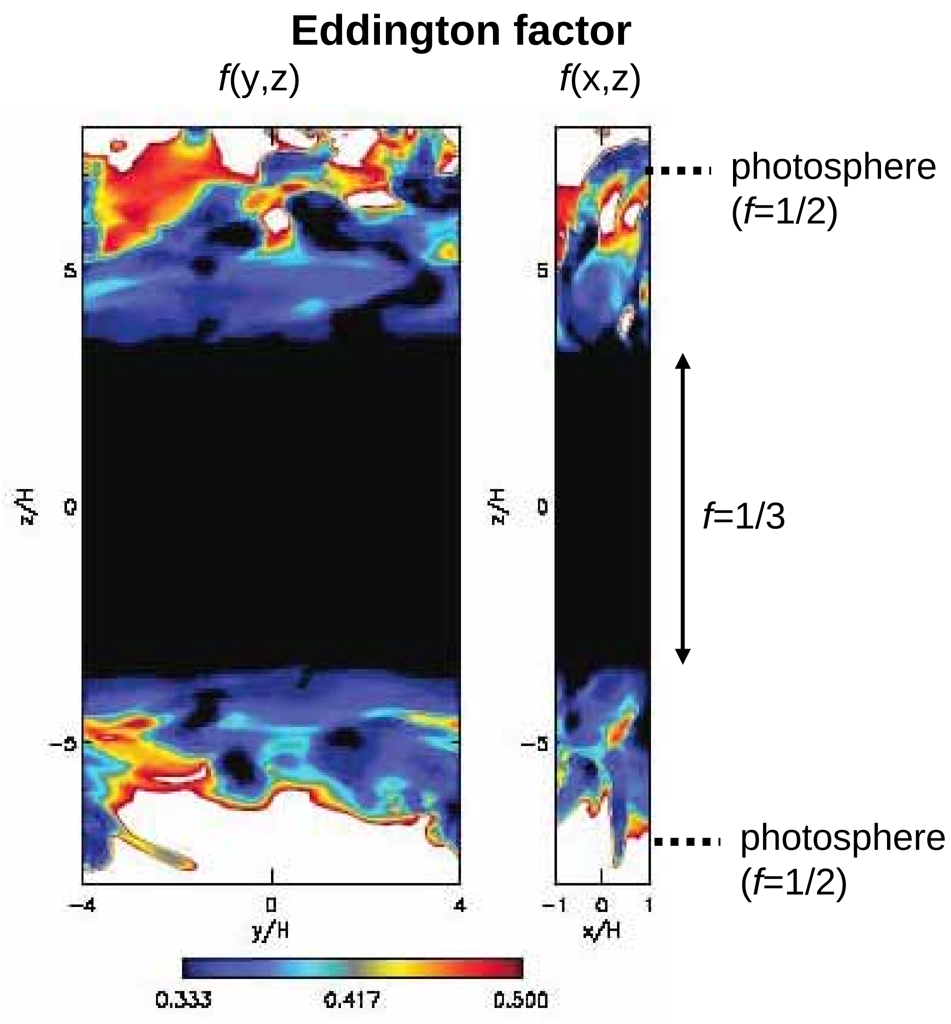

- Dissipated energy is transferred to the disk surface by radiation diffusion.

## Temperature

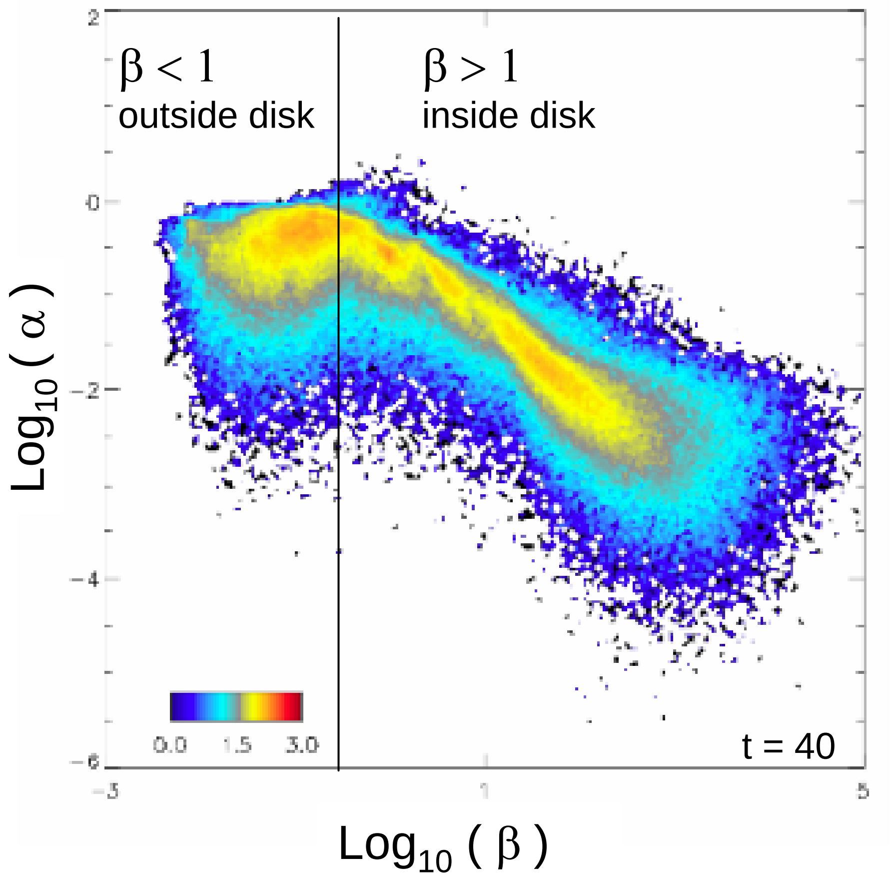



Gas and radiation well couples inside the disk body.

## Location of Photosphere



# Alpha Value




$\beta = P_{gas} / P_{mag}$

$\alpha = T_{r\phi} / P_{total}$

- disk body ( $\beta > 1$ )
  $\alpha \sim 0.3$
- disk atmosphere ( $\beta < 1$ )
  $\alpha \sim \beta^{-1}$

$\alpha_{mag} = T_{r\phi} / P_{mag}$

$\alpha_{mag} \sim 0.3$ in the entire region

# Summary

We calculate the vertical structure of disks where heating by dissipation of MRI-driven MHD turbulence is balanced by radiative cooling.