Relativistic Magnetic Reconnection

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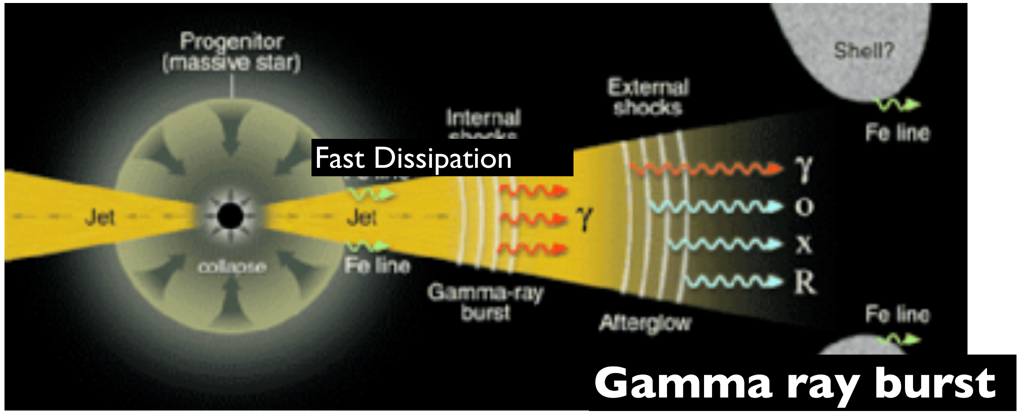
1. Introduction

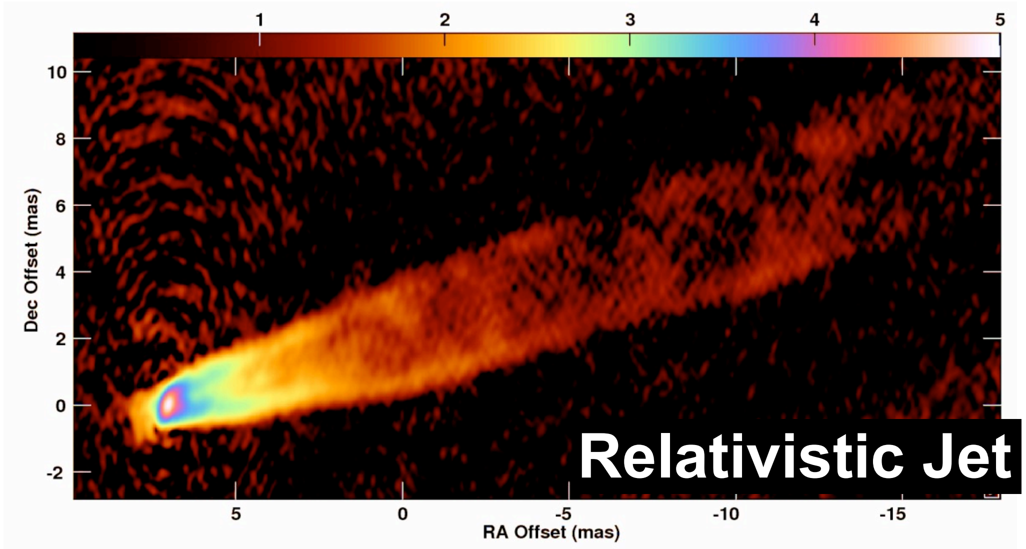
——"MR in High-Energy Astrophysics"

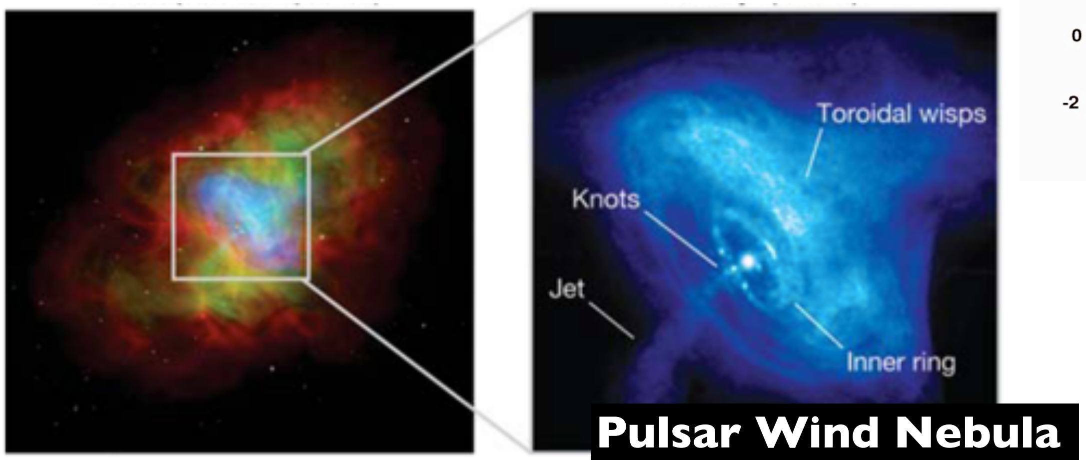
2. Relativistic MHD

3. Effects of Turbulence

1.1. MR in High-Energy Astrophysical Phenomena



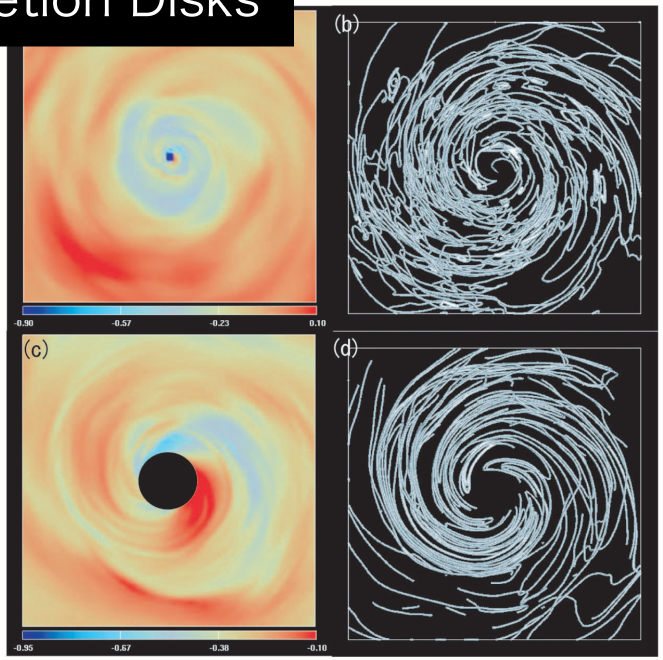


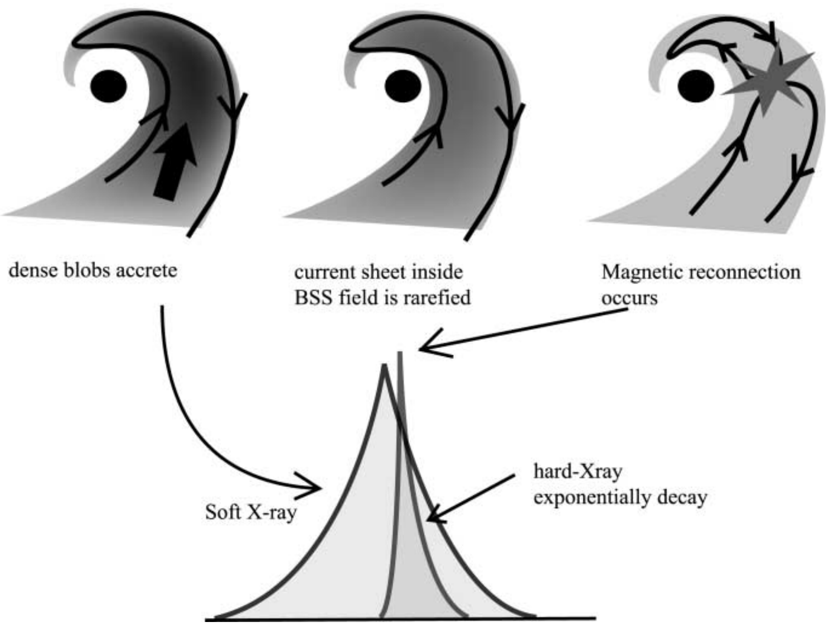


1.2. MR in Accretion Disks & Jets

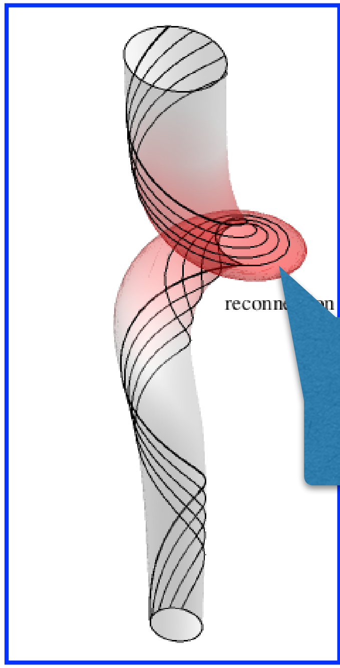
ref) Machida&Matsumoto (2003), ApJ 585, 429.

Accretion Disks

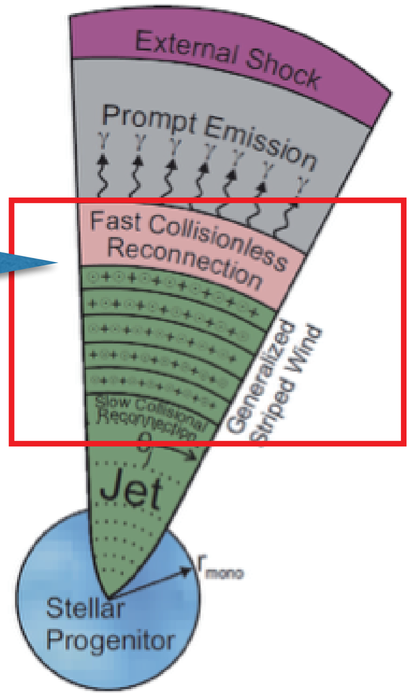




Jets



kink-turbulence



successive current sheets

1.3. Poynting-Dominated Plasma

E=-v/c×B, v⊥B

$$\boxed{\sigma \equiv \frac{\mathbf{E}\times\mathbf{B}c}{4\pi w c^2\gamma^2 v} = \frac{B^2}{4\pi w c^2\gamma^2}}$$

if σ > 1 : Poynting-Energy dominated ($c_A \sim c$)

< 1 : Matter-Energy dominated

$k_BT/Mc^2 \ll 1$

$$\Rightarrow \boxed{\sigma = \frac{B^2}{4\pi w c^2\gamma^2} = \left(\frac{M}{4\pi n e^2}\right)\left(\frac{eB}{Mc}\right)^2 = \left(\frac{\omega_B}{\omega_p}\right)^2}$$

if σ > 1 : Gyro-rotation is faster than plasma-oscillation

if reconnection **($\gamma \sim 1$, $k_BT \ll Mc^2$)**

proton

$$\Rightarrow \boxed{\sigma = 2\left(\frac{B^2/8\pi}{\rho c^2}\right) \sim 0.53\ \left(\frac{B}{0.1[\mathrm{G}]}\right)^2\ \left(\frac{n}{1[\mathrm{cm}^{-3}]}\right)^{-1}}$$

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2.1 Magnetic Reconnection

ref) Sweet, (1958)
Parker, (1957; 1963)

<u>Assumptions</u>: steady flow and uniform resistivity
(Sweet-Parker model)

R[illegible]ction rate:

u_{in} [illegible] $/ S$

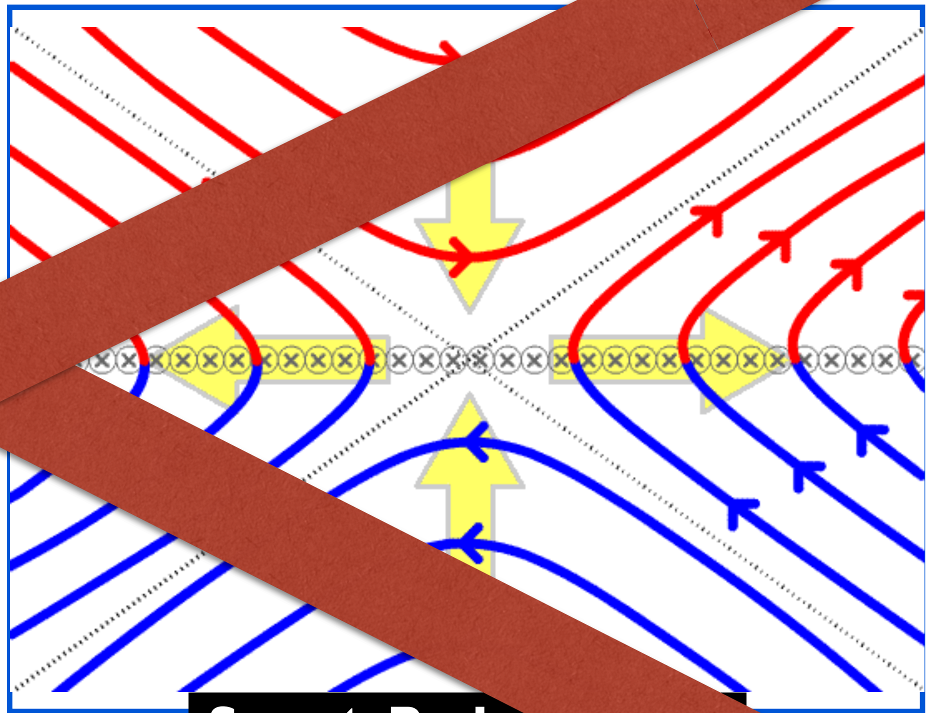
$S = L\, c_A$ [illegible]

In many astrophysical ob[illegible]

$S = L\, c_A / \eta$ [illegible] > 1

[illegible] $_A / \sqrt{S} << c_A$

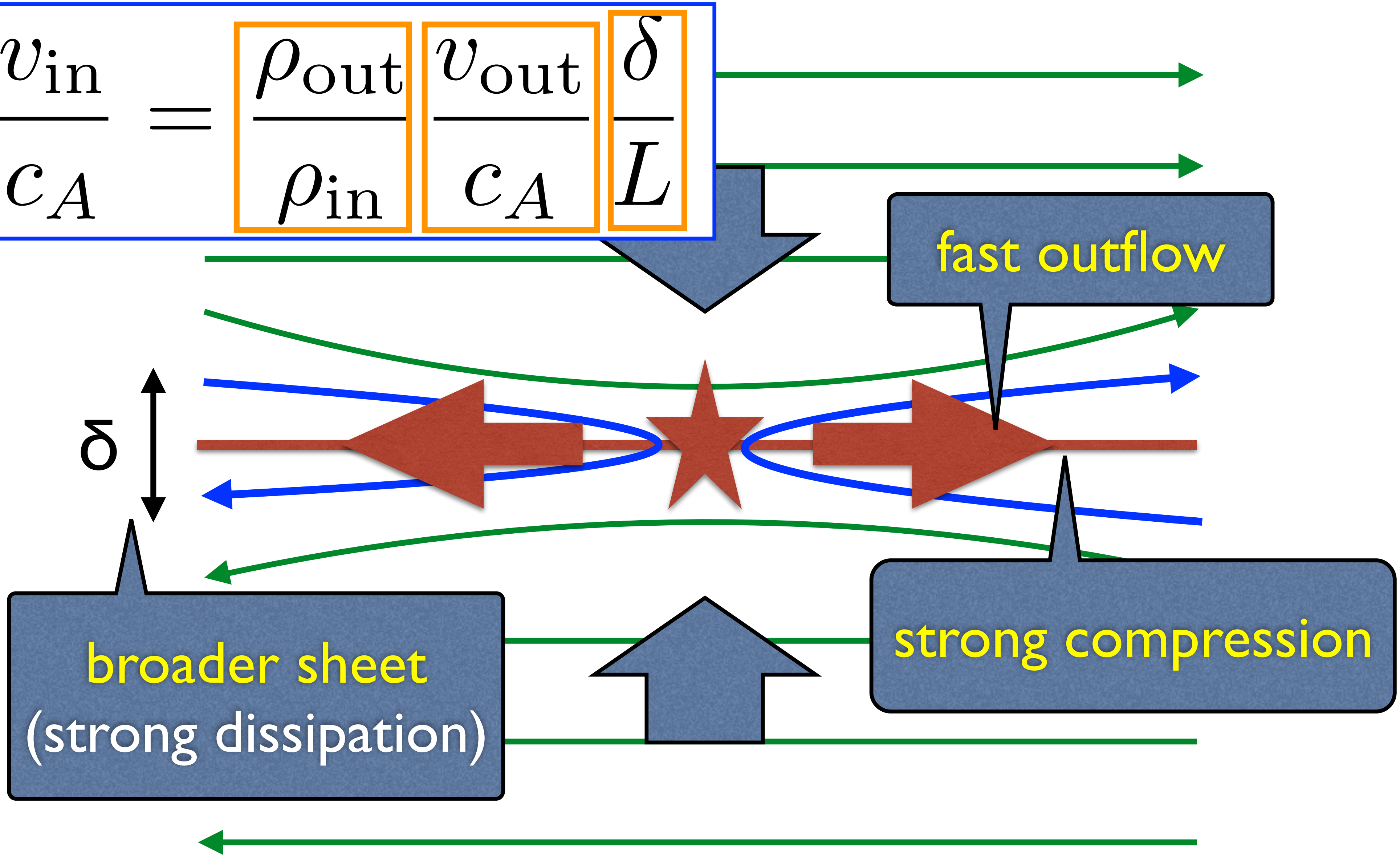
very slow ....



$\delta/L = 1/\sqrt{S}$ ⟷ too thin…

2.2. Conditions for Fast Reconnection

$$\frac{v_{\rm in}}{c_A} = \boxed{\frac{\rho_{\rm out}}{\rho_{\rm in}}}\,\boxed{\frac{v_{\rm out}}{c_A}}\,\boxed{\frac{\delta}{L}}$$



2.3. Relativistic Effects on Reconnection

ref) Lyutikov&Uzdensky 2003, ApJ 589, 893
Lyubarsky, (2005), ApJ, 358, 113.
Zenitani etal, (2009), ApJ 696, 1385.

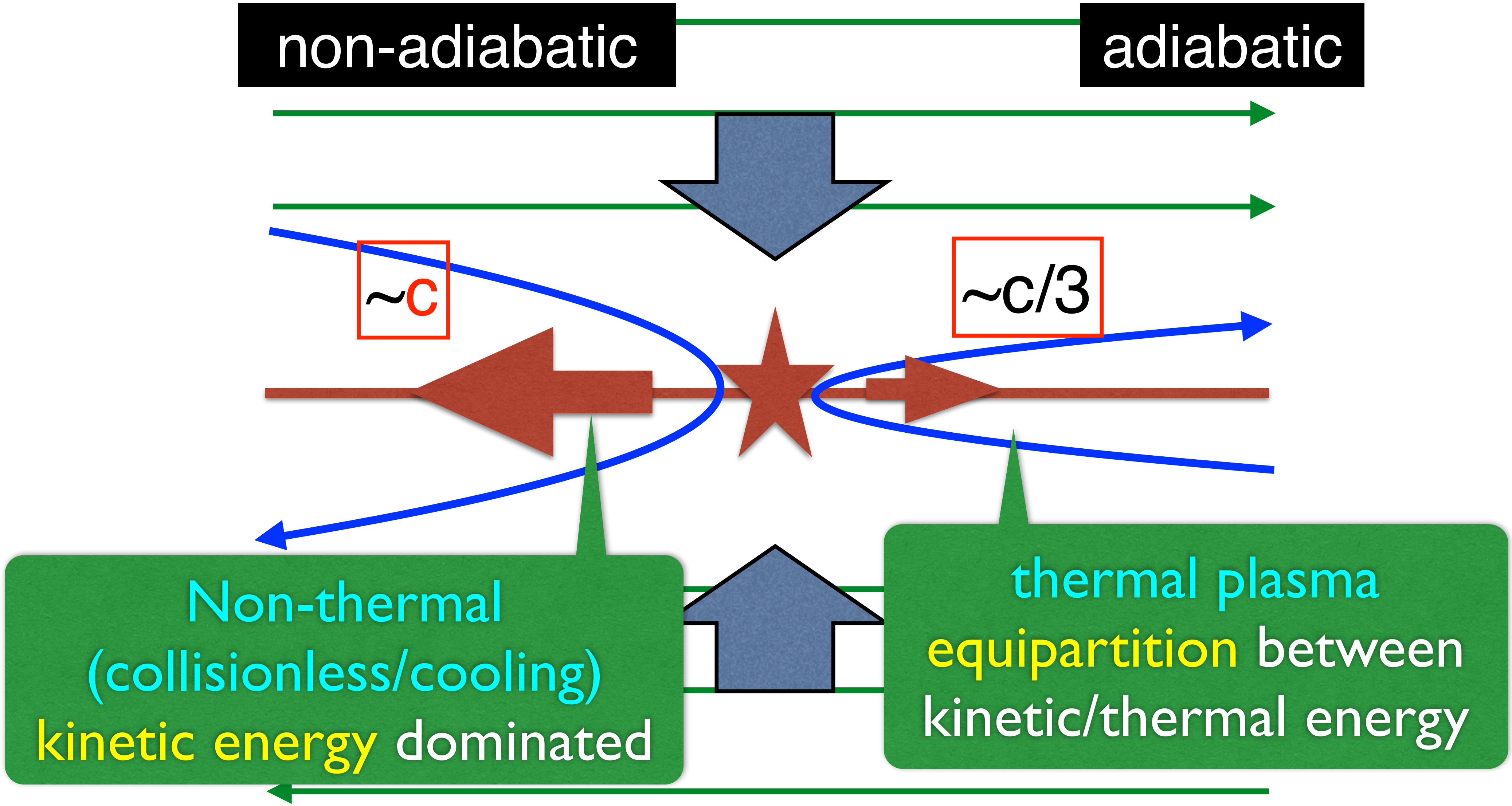
density=ρ_0 γ
transfer more matter
elocity

Maximum SP Rate

$$\frac{\gamma_{\rm in} v_{\rm in}}{\gamma_{\rm A} c_{\rm A}} \simeq \sqrt{\frac{2}{S_{\rm L}}} \left(\frac{\sqrt{c_{\rm A}}}{\gamma_{\rm A}} \right) \boxed{\gamma_{\rm out} \sqrt{v_{\rm out}}}$$

$k_B T/mc^2 > 1, \gamma > 1$
= large inertia
strong beaming
=>decelerate…
thin sheet…

2.4. Reconnection Jet Acceleration



1. Introduction

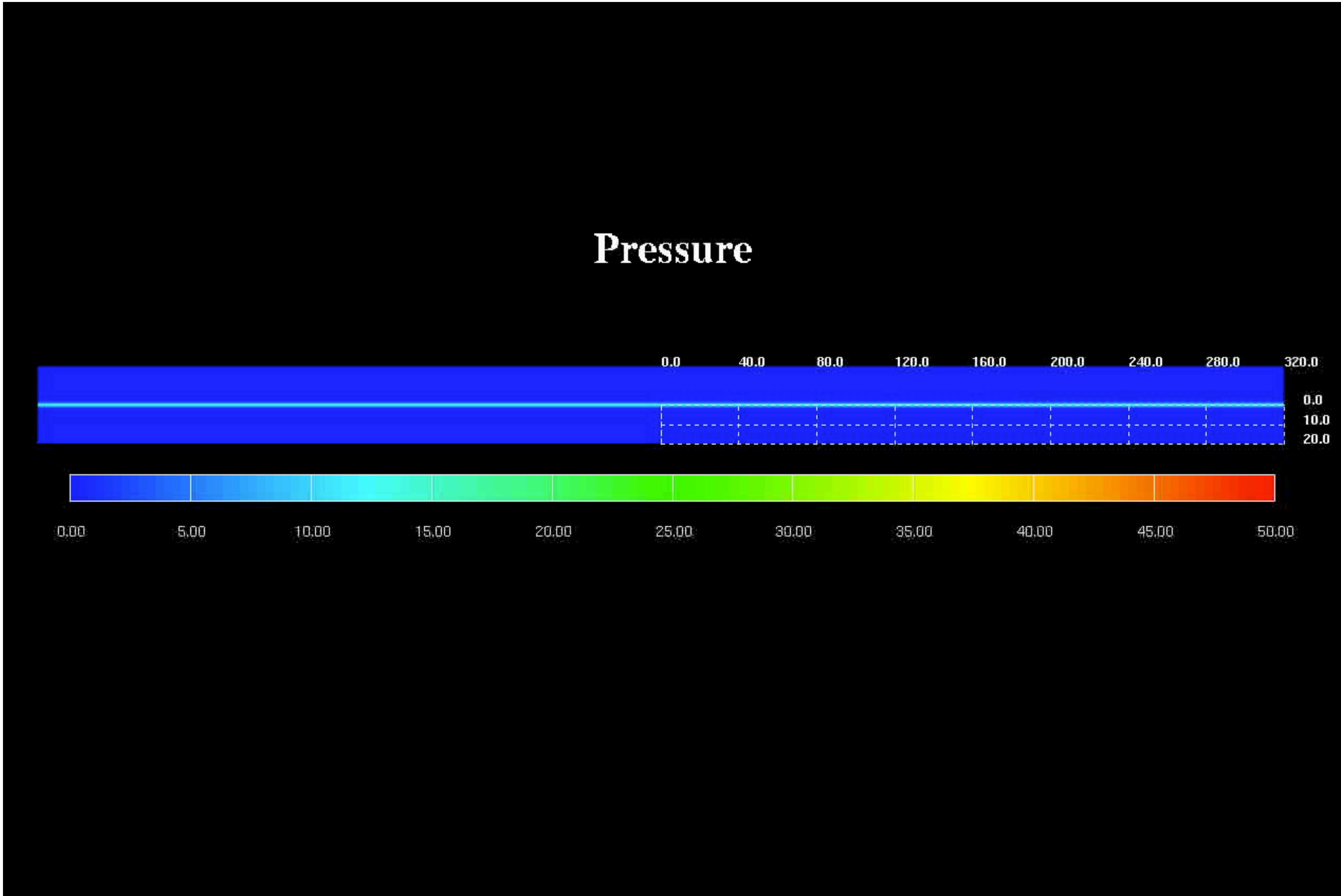
——“MR in High-Energy Astrophysics”

2. Relativistic Reconnection

3. Effects of Turbulence

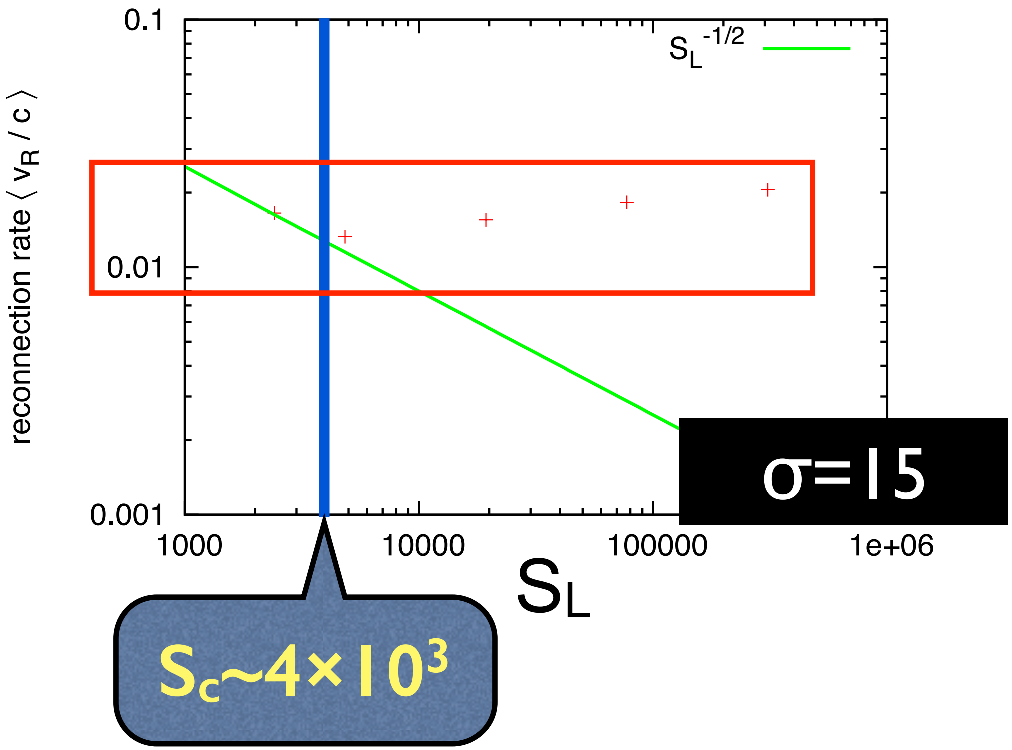
3.1. Plasmoid-Chain

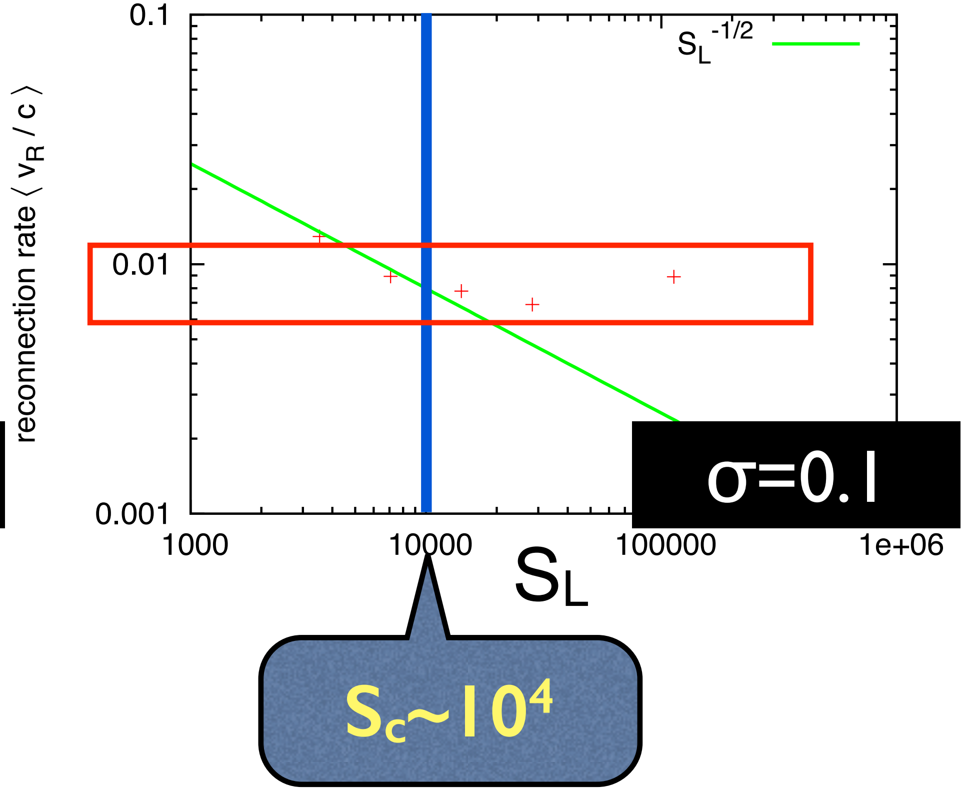
ref) Shibata & Tanuma, 2001, EPS, 53, 473
Loureiro et al. 2007, Phys. Plasmas 14, 100703
Bhattacharjee et al., 2009, Phys. Plasmas 16, 112102
Takamoto, 2013, ApJ 775, 50.



3.2. Lundquist Number Dependence

ref) M.Takamoto, 2013, ApJ, 775, 50.





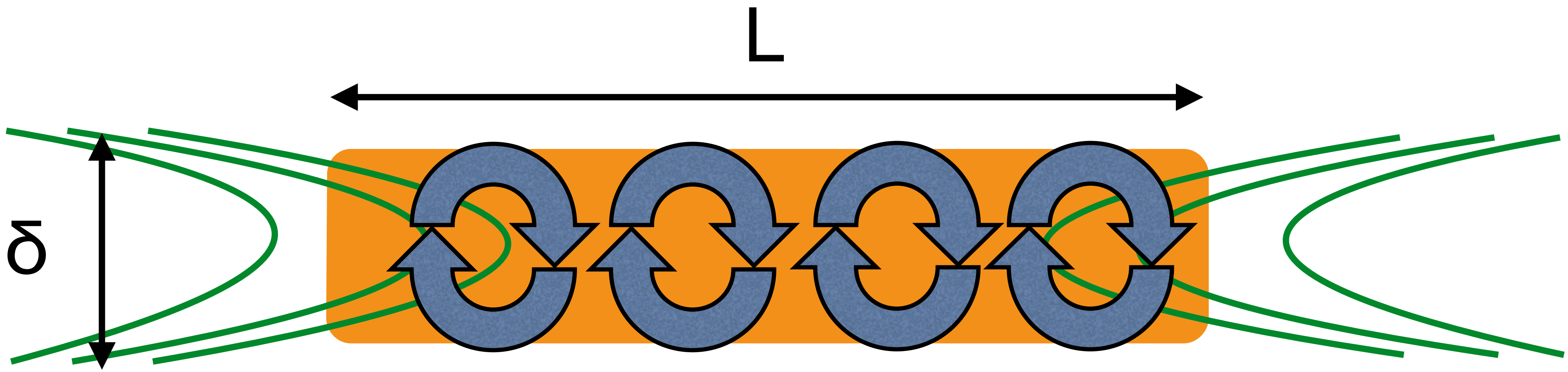
Reconnection Rate becomes
independent of Lundquist number S_L

when S_L > $S_{L,C}$: critical value
at which Plasmoid instability occurs

3.3. 3D Turbulent Sheets

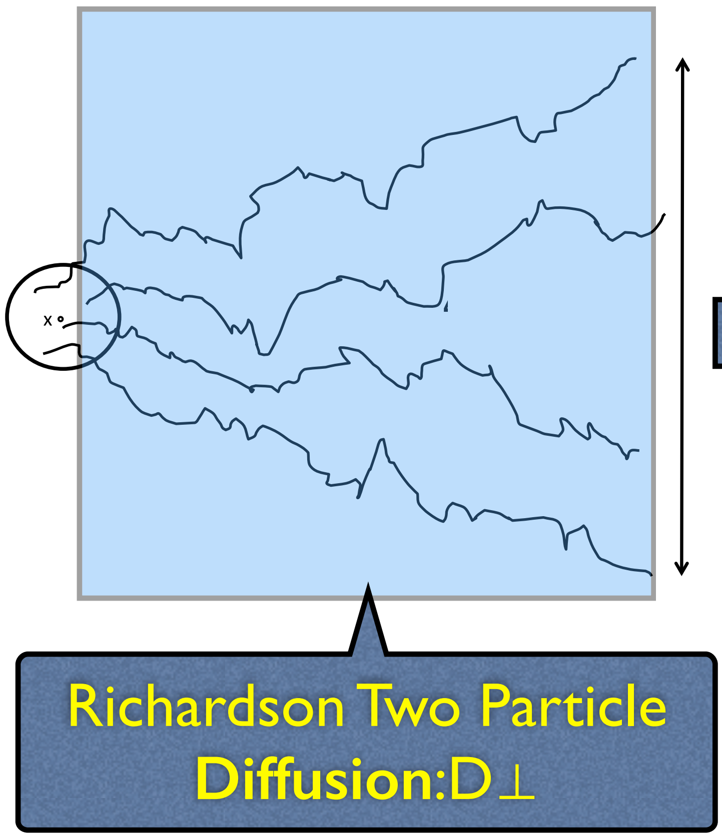
ref) Lazarian & Vishniac (1999), ApJ, 517,700
Higashimori+, (2013), PRL 110, 255001

$$\boxed{\frac{v_{\rm in}}{c_A} = \frac{\rho_{\rm out}}{\rho_{\rm in}} \frac{v_{\rm out}}{c_A} \boxed{\frac{\delta}{L}}}$$

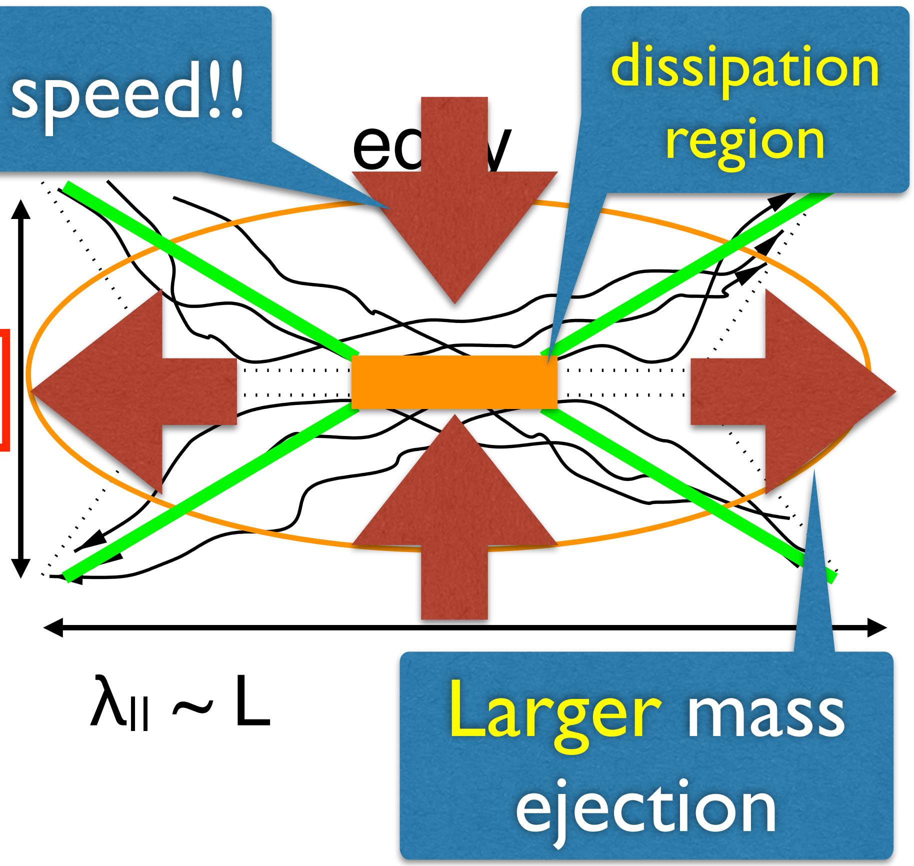


3.4. Theoretical Explanation

ref) Lazarian & Vishniac (1999), ApJ, 517, 700
Eyink, Lazarian, Vishniac, (2011), ApJ, 743, 51.



$\lambda_{\perp} \sim \delta$

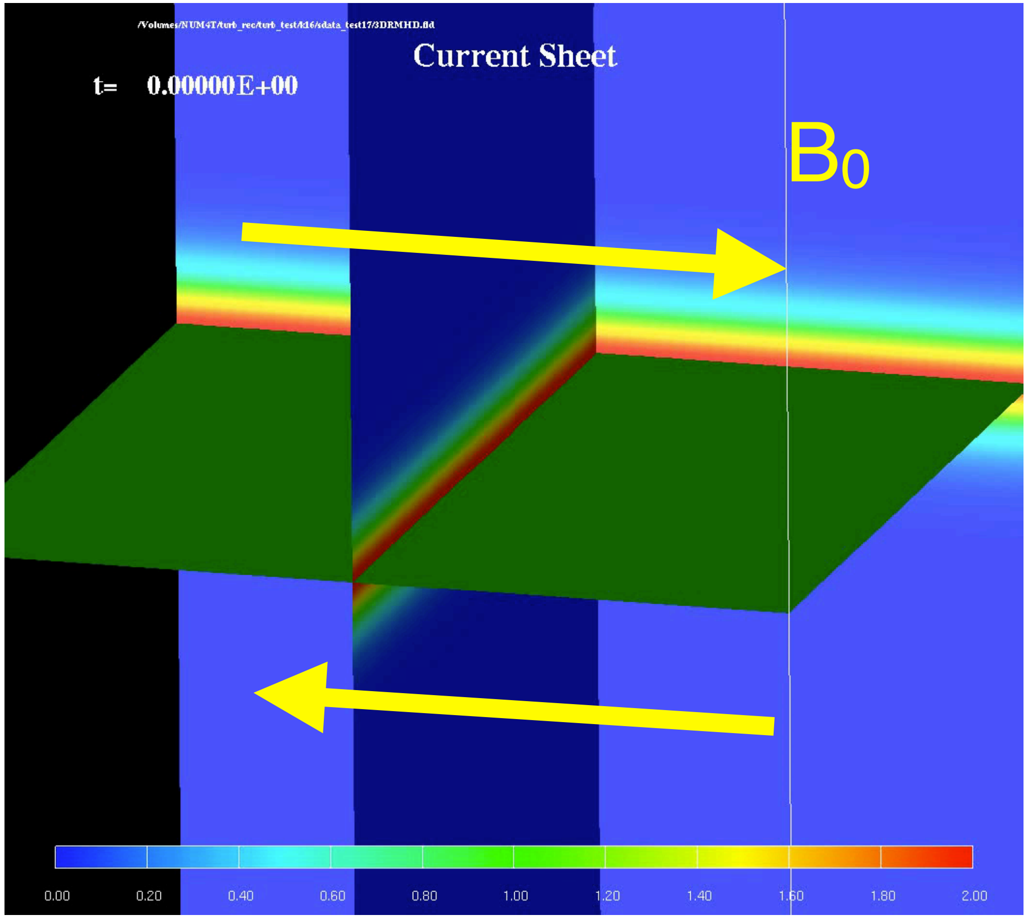


$$\frac{\delta}{L_x} = \boxed{M_A^2} \min\left\{ \left(\frac{L_x}{L_i}\right)^{1/2}, \left(\frac{L_i}{L_x}\right)^{1/2} \right\}$$

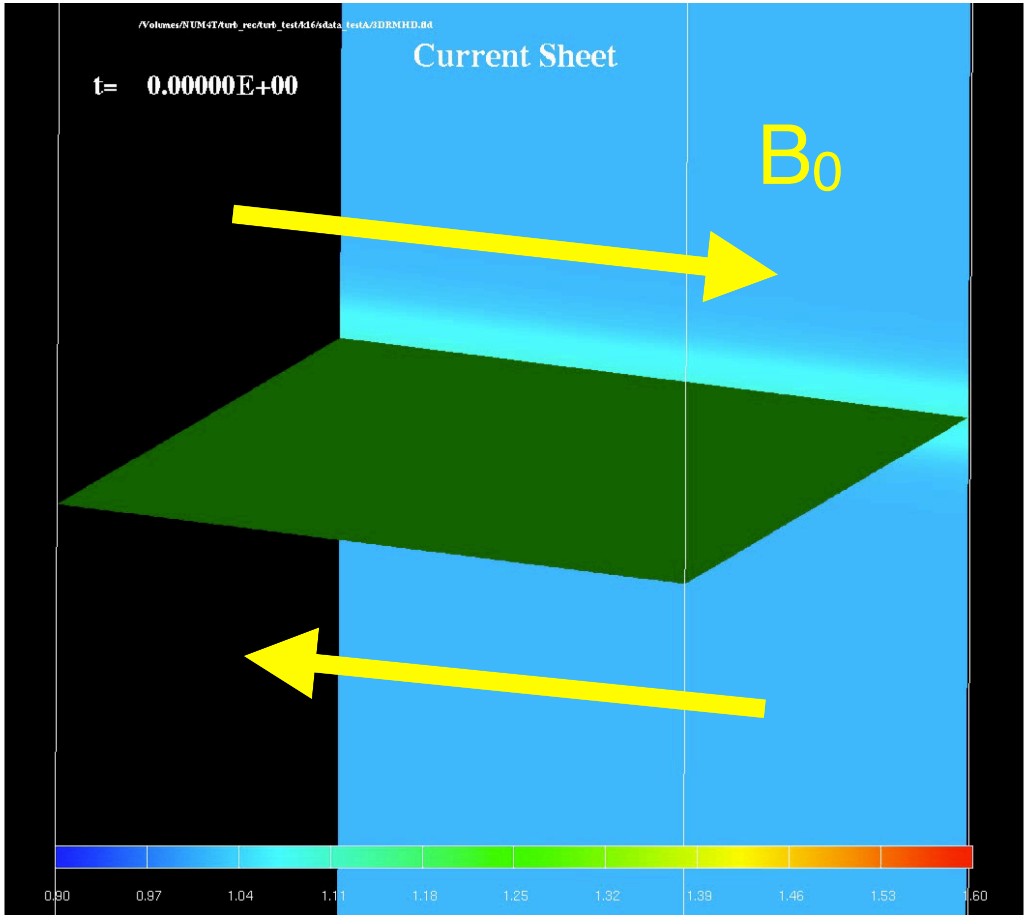
3.5. Relativistic Turbulent Reconnection

ref) Takamoto+ (2015), ApJ, 815, 16.

Poynting Dominated($\sigma = 5$)

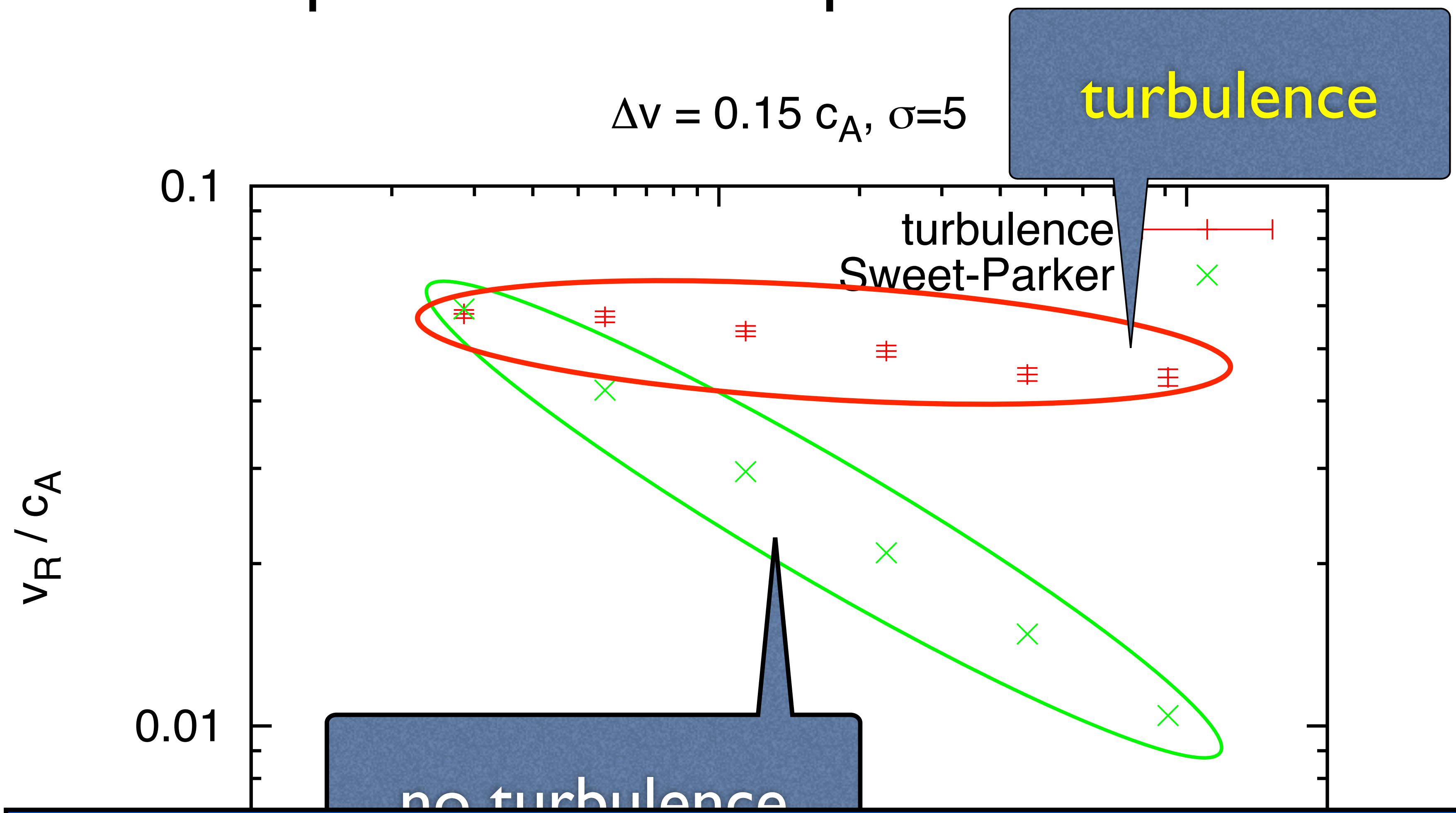


Matter dominated($\sigma = 0.04$)



- $k_B T/mc^2 = 1$
- **driven** turbulence
  injected around central region

3.6. Lundquist Number Dependence



fast & resistivity independent
mechanism

3.7 Necessary Turbulence Energy in Poynting-Dom.

3-Mach Number

if we set: $$\boxed{\frac{v_{\rm turb}}{c_{\rm A}} \equiv \alpha}$$

$$\boxed{\frac{\epsilon_{\rm turb}}{\epsilon_{\rm B}} \equiv \frac{\rho_0 h v_{\rm turb}^2/2}{B_0^2/8\pi} = \frac{\alpha^2}{1\boxed{+\sigma}}}$$

$$\left(c_{\rm A} \equiv c\sqrt{\frac{\sigma}{1+\sigma}}, \quad \sigma \equiv \frac{B^2}{4\pi\rho_0 h c^2\gamma^2} = \frac{B_0^2}{4\pi\rho_0 h c^2}\right)$$

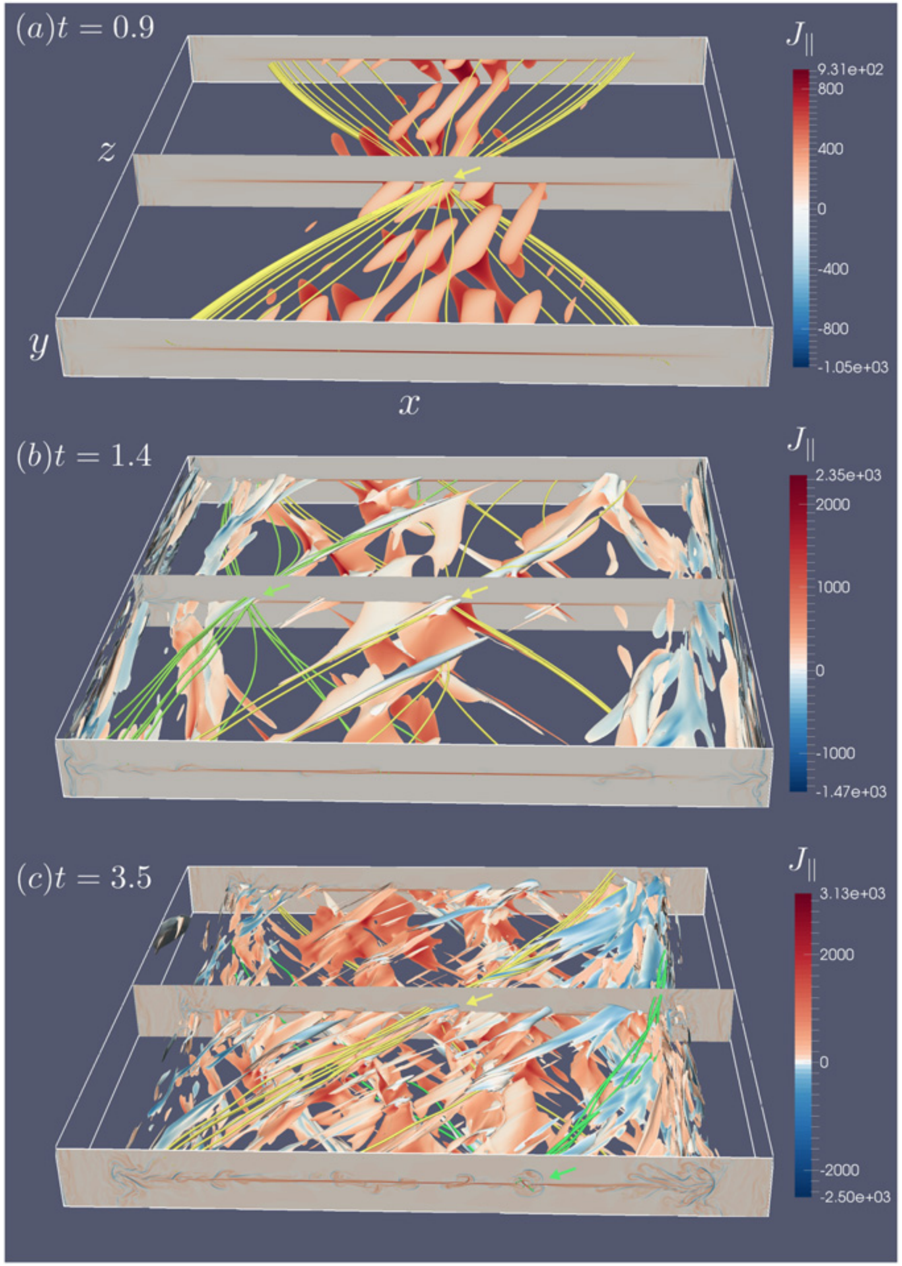
if we assume: $v_{\rm turb}/c_{\rm A}= 0.3$, $\sigma = 10$,

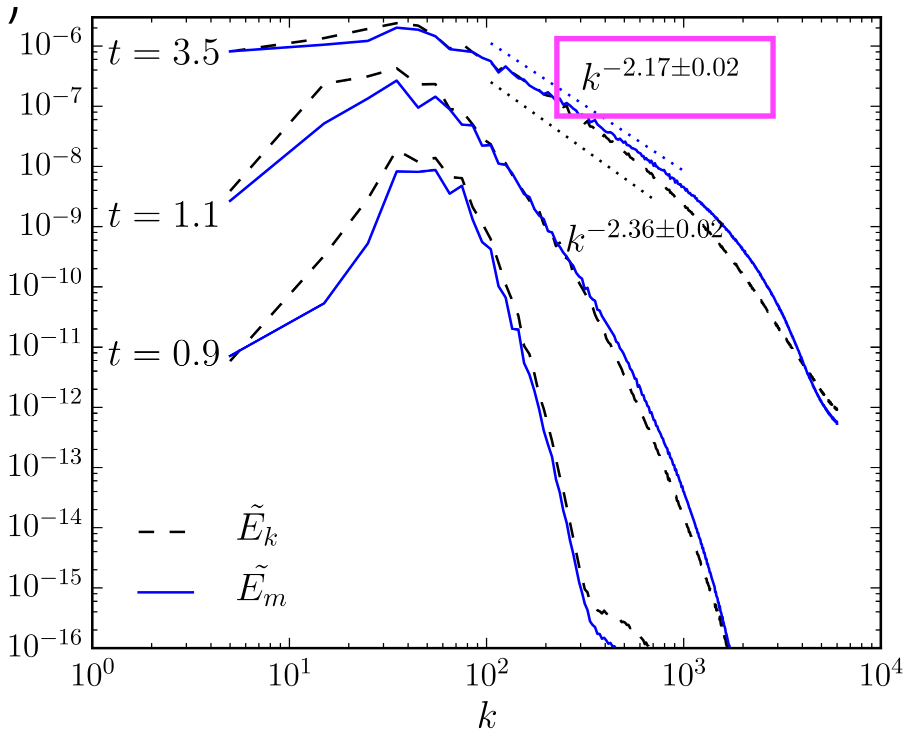
$\varepsilon_{\rm turb} /\varepsilon_{\rm B} \sim 0.01$

just 1% of magnetic field energy is sufficient!!

3.8. Self-Generated Turbulence in Sheets?

ref) Huang & Bhattacharjee (2016), ApJ 818

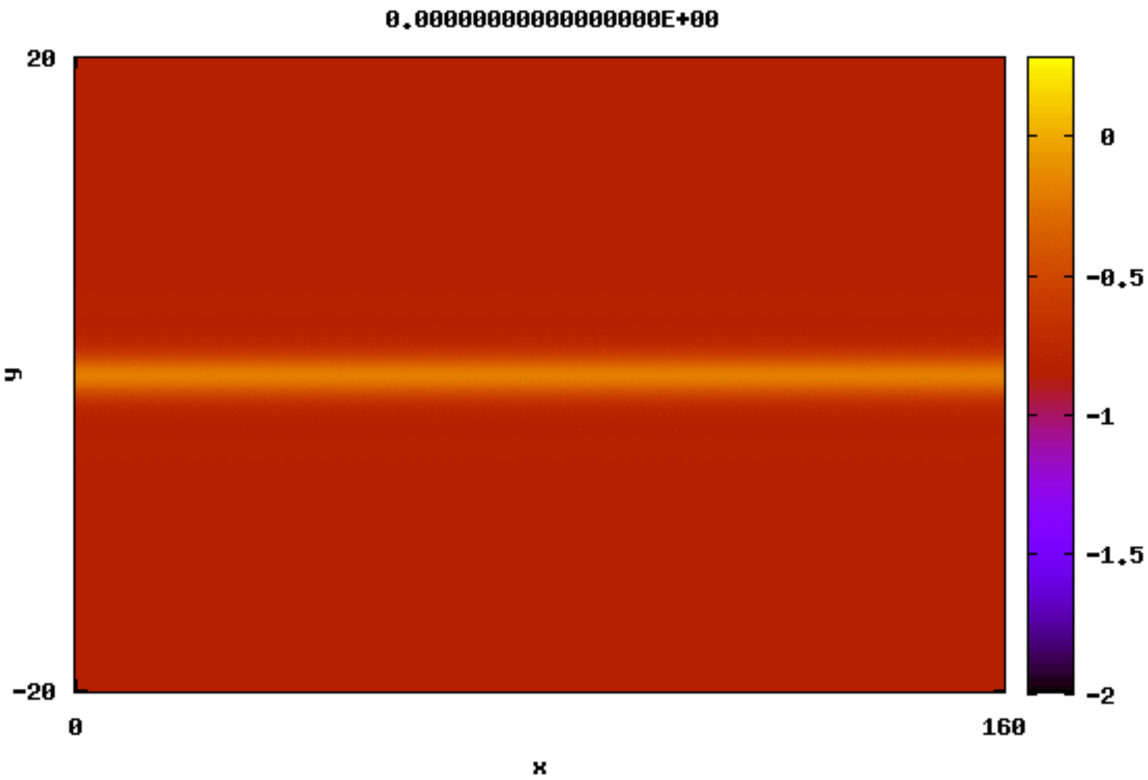




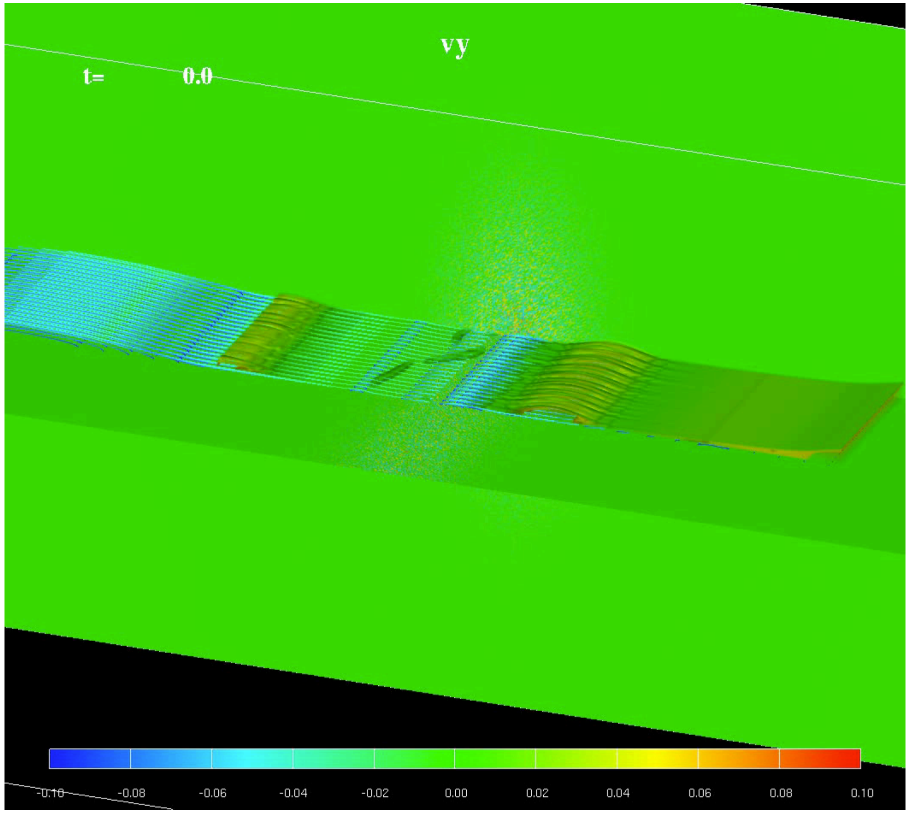
different turbulent law
caused by
different injection mechanism
and non-trivial background?

3.9. Relativistic Self-Generated Turbulence in Sheets

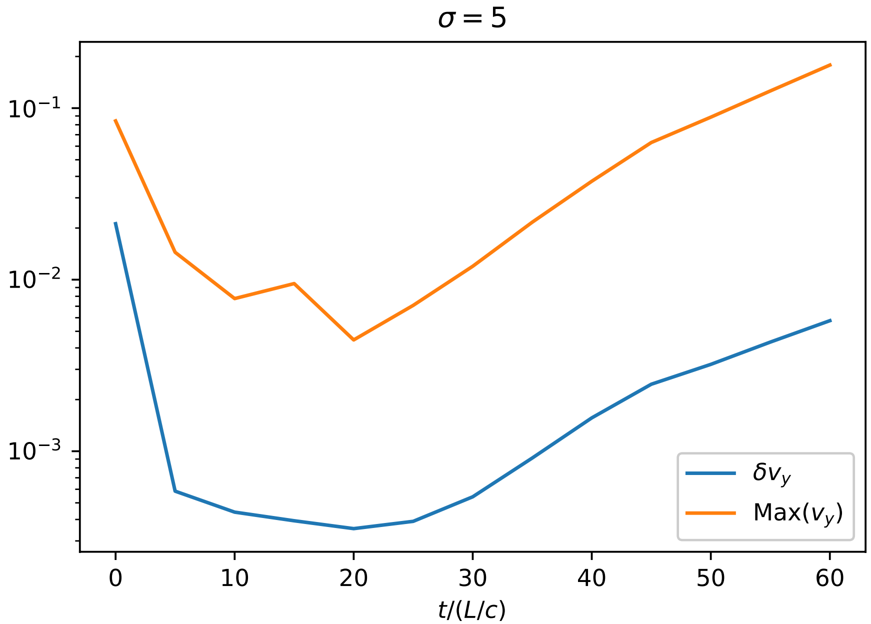
2D simulation

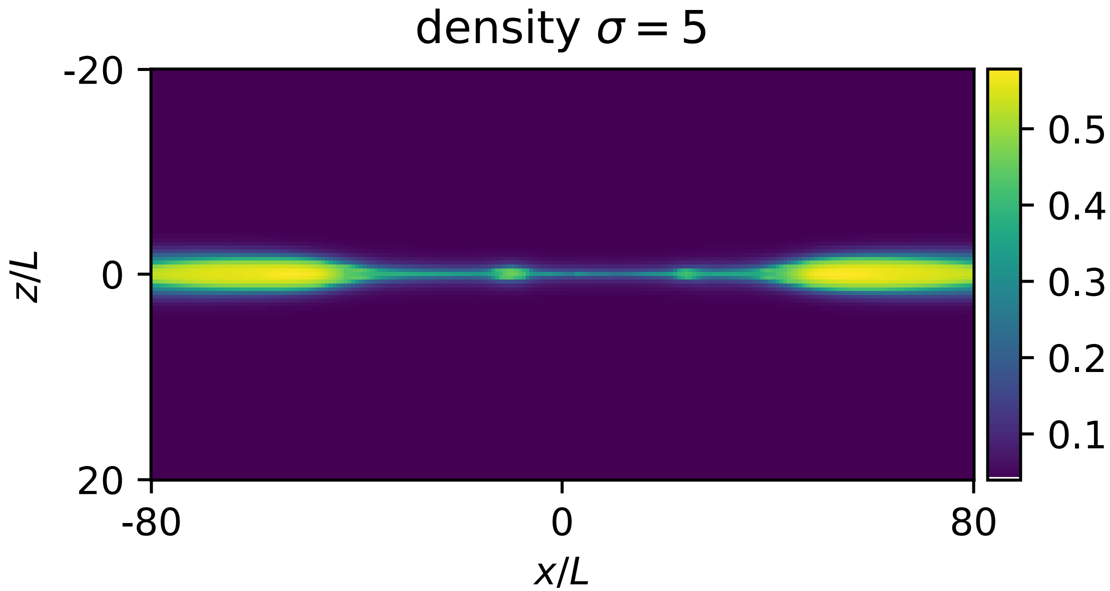


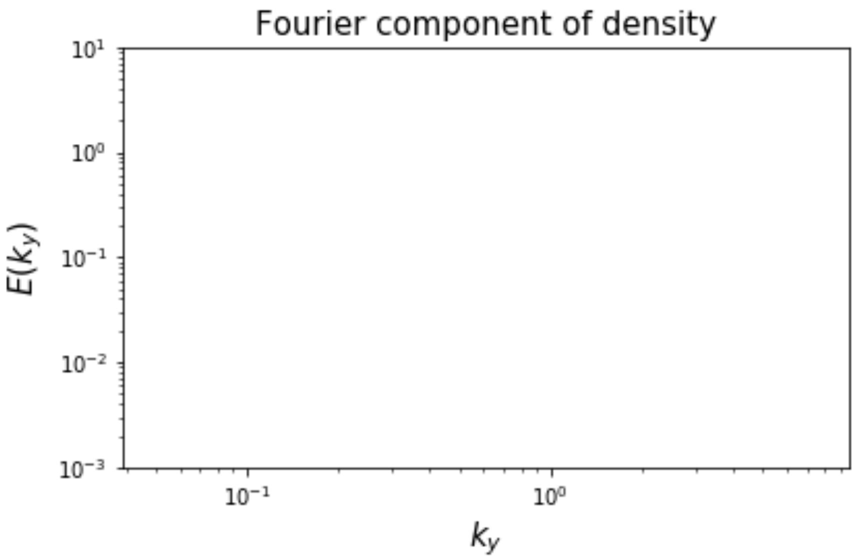
3D simulation

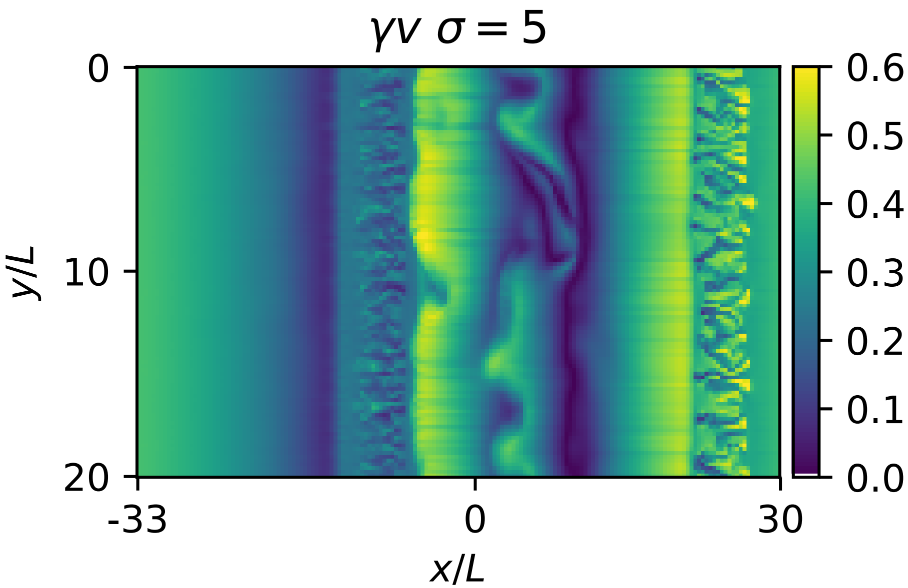


3.10. some results









3.11. Model of MR in global 3D simulation

if MHD

if 2D & $S>10^4$

if 3D & strong turbulence ($v_{turb}>0.1c_A$)

if 3D & $\sigma > 1$ & high S

⇒ ideal MHD

high-resolution

if collisionless

⇒ new formalisms (cosmic-ray, anisotropy, PIC, etc)

if strong cooling or radiation dominated

⇒ resistive RMHD with proper resistivity

Summary

- We investigated various kinds of relativistic reconnection in Poynting-dominated plasma

- We found that the reconnection rate is highly enhanced (dissipation time ~ **20 - 30 Alfvén crossing time**)

  —Turbulent Reconnection : $v_R/c_A \sim 0.05$

- Reconnection rate becomes **independent** of the Lundquist number (resistivity)

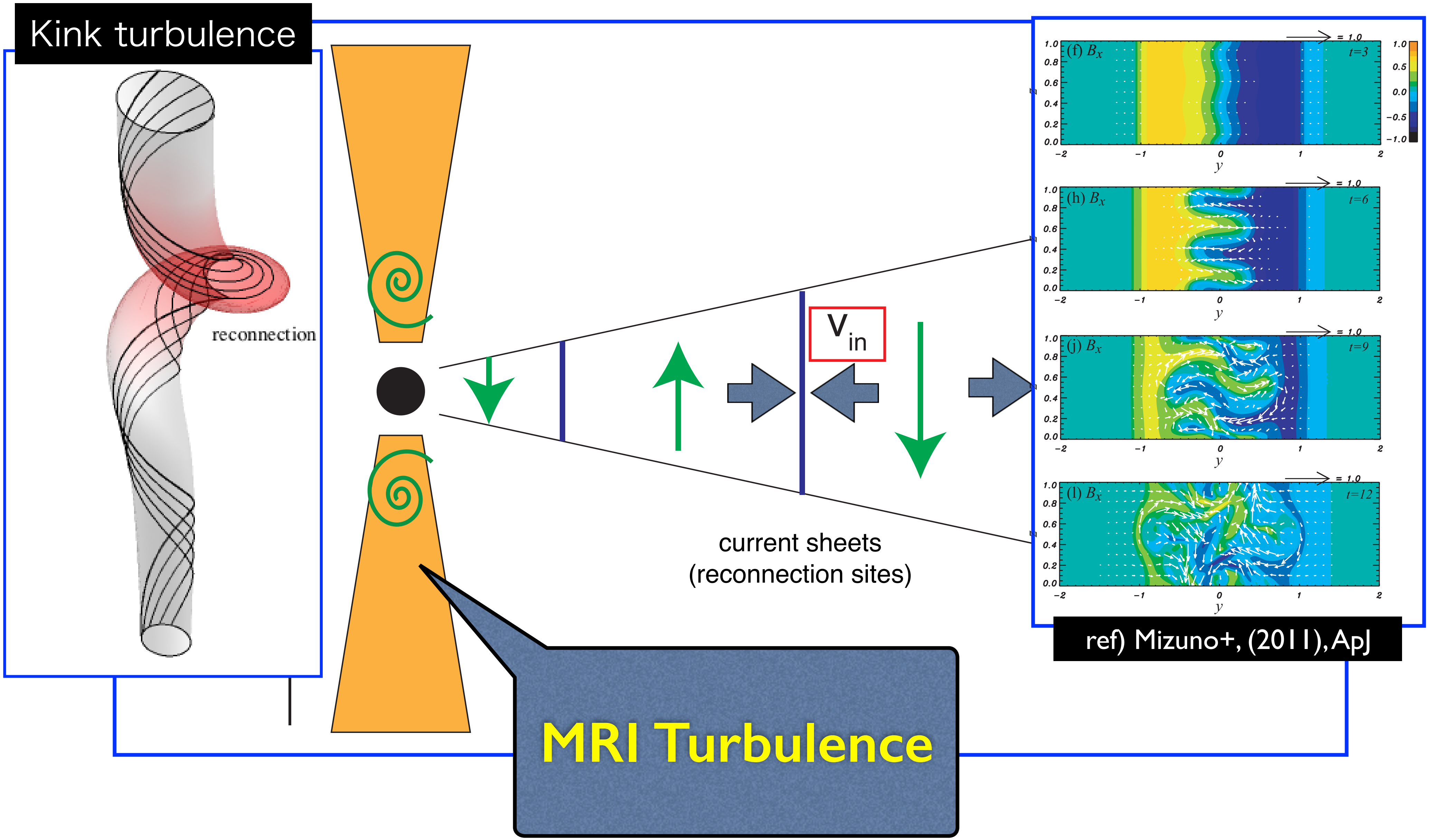
- **Only 1%** of magnetic field energy is sufficient to drive turbulent reconnection in high-σ jets!

- Compressible effect becomes important in high-σ plasma.

O Ma Ke

Relativistic Jets

ref) MT+,(2015), ApJ, 812, 15
Rieger & Aharonian, (2012), MPLA 27, 30030



2.1. Relativistic Magnetohydrodynamics

Basic equations of RMHD:

$$
\left\{
\begin{aligned}
&\partial_t(\rho\boxed{\gamma}) + \partial_i(\rho\boxed{\gamma} v^i) = 0, \\
&\partial_t(\rho\boxed{h_{tot}\gamma^2} v^j - \boxed{b^0 b^j}) + \partial_i(\rho\boxed{h_{tot}\gamma^2} v^i v^j + p_{tot}\delta^{ij} - b^i b^j) = 0, \\
&\partial_t(\rho\boxed{h_{tot}\gamma^2} - p_{tot} - \boxed{(b^0)^2}) + \partial_i(\rho\boxed{h_{tot}\gamma^2} v^i - \boxed{b^0 b^i}) = 0, \\
&\partial_t B^j + \partial_i(v^i B^j - B^i v^j) = 0, \quad \partial_i B^i = 0. \\
&\qquad h_{tot} = 1 + \epsilon + \boxed{\frac{b^2}{\rho}}, \quad p_{tot} = p_{gas} + \frac{b^2}{2}
\end{aligned}
\right.
$$

<u>features:</u>

- correction from Lorentz factor and inertia of energy
- tension and pressure from magnetic field

2.2. Relativistic Effects:

Lorentz contraction:

lab frame density: $\rho \Rightarrow \rho \boxed{\gamma}$

larger density

Alfven velocity:

$$c_A/c = \frac{B}{\sqrt{4\pi\rho h \boxed{+ B^2}}} < 1$$

sub-luminal

Electric Field:

$$\boxed{qE} \sim (\nabla E)E \sim v^2 B^2/R \sim \boxed{p_B \mathbf{v} \cdot \nabla \mathbf{v}}$$

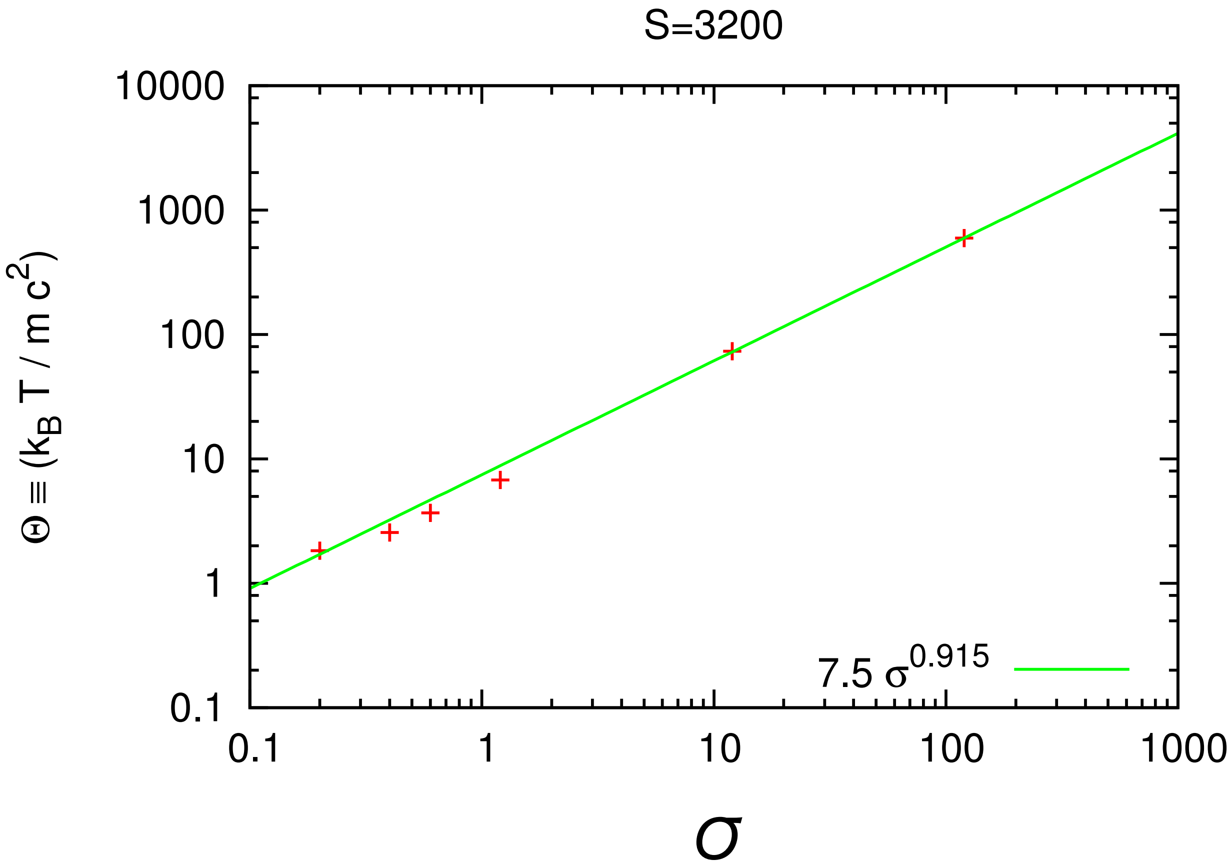
$$j \times B \sim (\nabla \times B \boxed{- \partial_t E}) \times B \sim (\nabla \times B - \partial_t v B) \times B$$

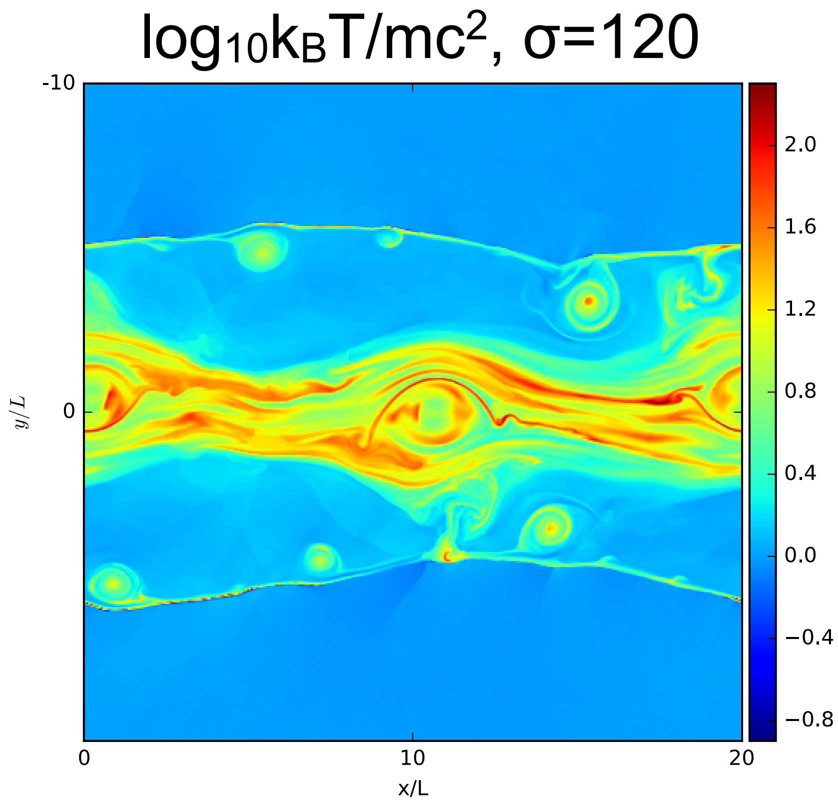
$$\sim (\nabla \times B) \times B \boxed{- p_B \partial_t v}$$

inertia from magnetic field

2.5. DTM Temperature dependence

ref) Takamoto+(2015), MNRAS 454, 2972.



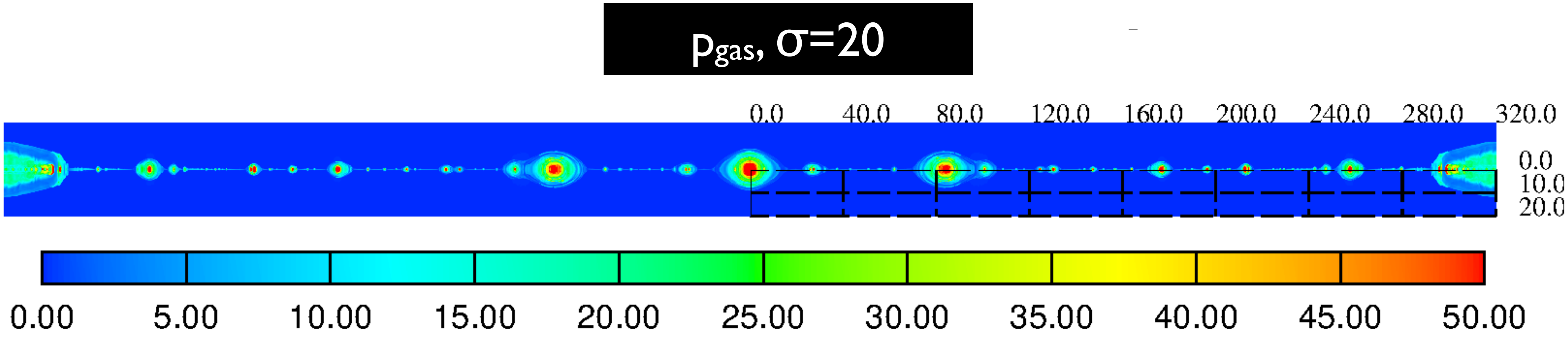


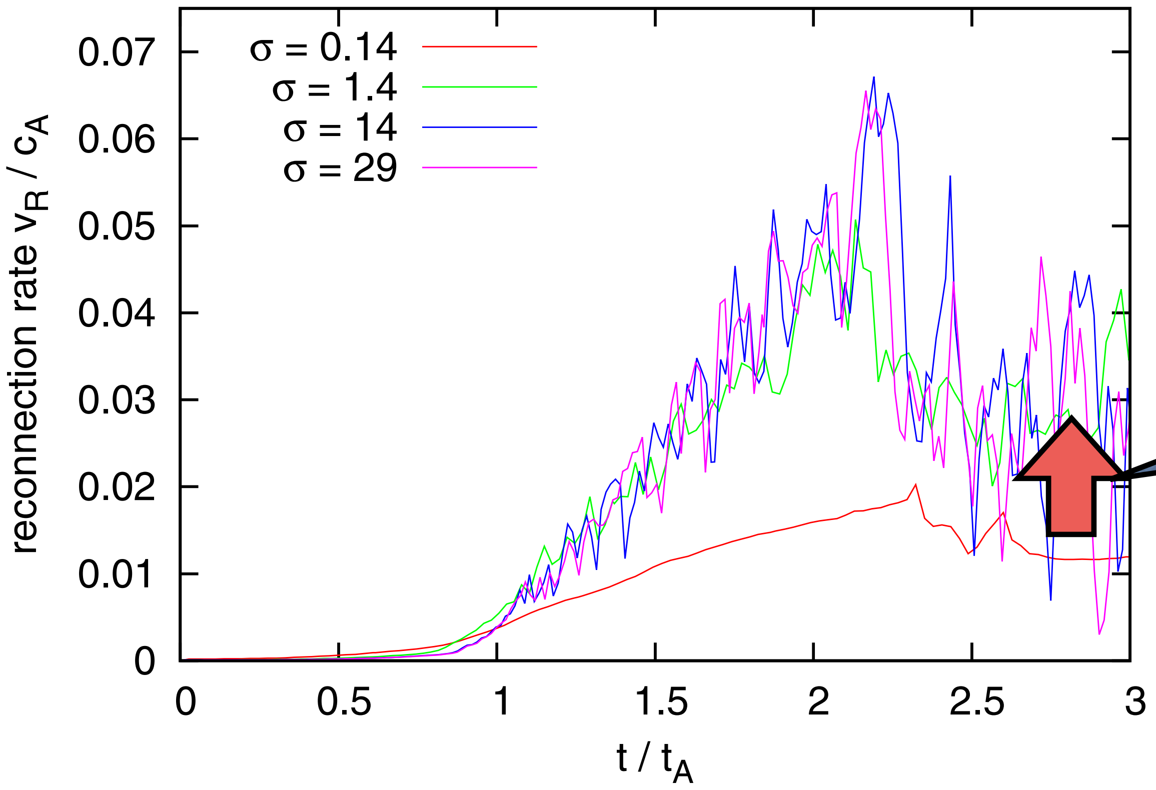
$$\boxed{\frac{k_B T}{mc^2} \simeq 7.5\sigma = 7.5 \frac{B^2}{4\pi w c^2 \gamma^2}}$$

linear in σ

2.4. Relativistic Plasmoid-Chain

ref) Takamoto, (2013), ApJ 775, 50.





reconnection rate increase with σ

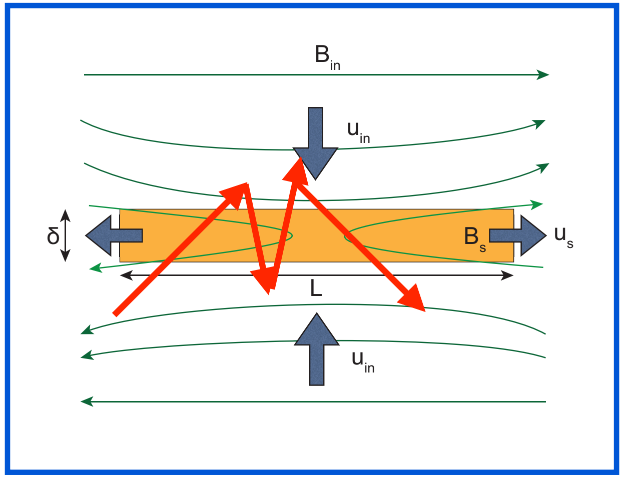
10. Particle Acceleration by Reconnection

- If Turbulent Reconnection:

➡ shock-like acceleration

ref) Pino&Lazarian, (2005), A&A 441, 845.

$$N(E) \propto E^{-2.5}$$



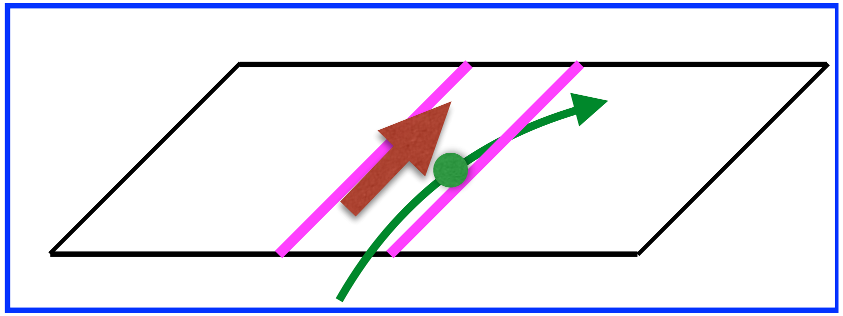
- X-point Acceleration:

➡ direct acceleration

by electric field at X-point

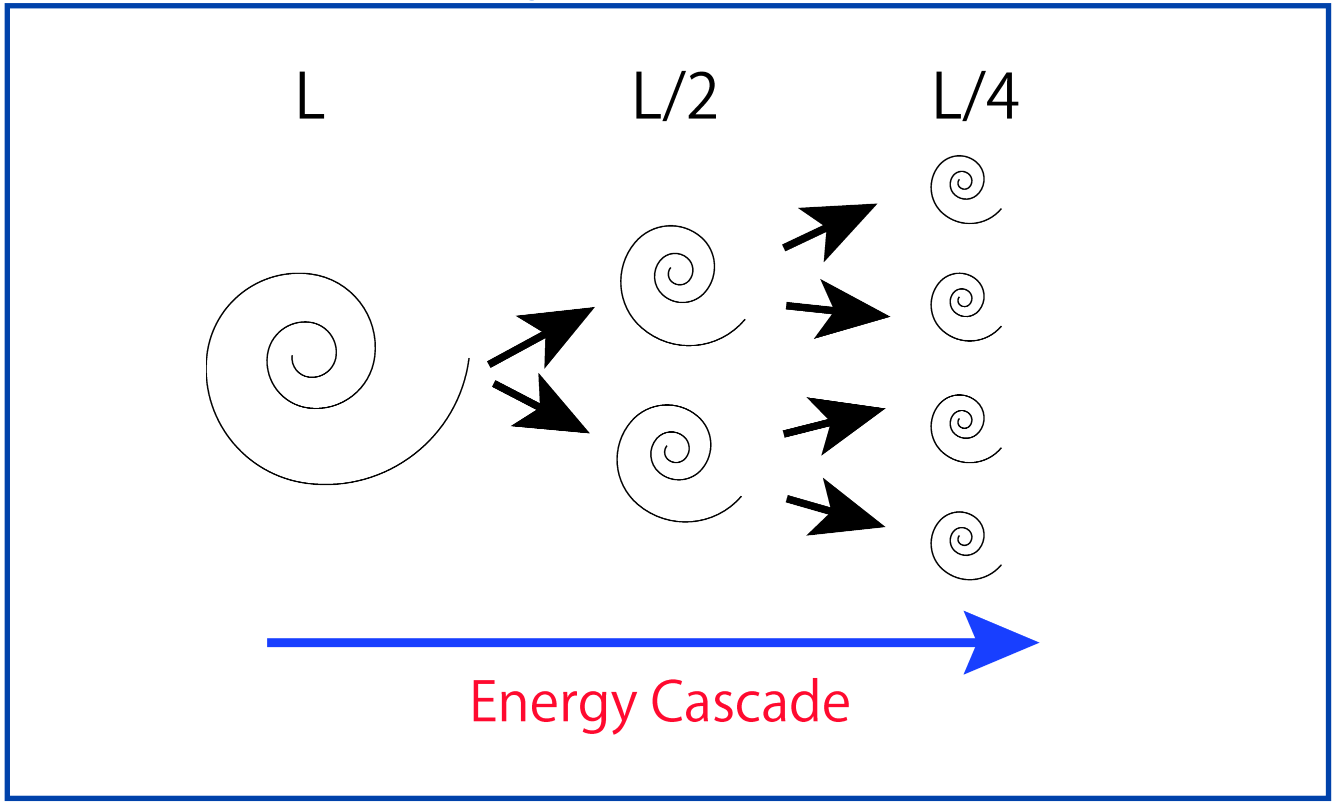
ref) Zenitani & Hoshino (2001), ApJL, 562, 63.
Bessho & Bhattacharjee (2012), ApJ, 750, 129.
Sironi & Spitkovsky, (2014), ApJL, 783, 21.

$$N(E) \propto E^{-1.4}$$



Kolmogorov Turbulence

Assumptions:
- Homogeneous and isotropic turbulence
- Steady state



$$v_l = v_L (l/L)^{1/3} \propto l^{1/3}$$:eddy velocity

$$E(k) \propto k^{-5/3}$$: Energy Spectrum

MHD Turbulence (Goldreich-Sridhar model)

<u>Assumptions:</u>
- Magnetic Field exists
- Steady state

Features: ·eddy is enlarged along B

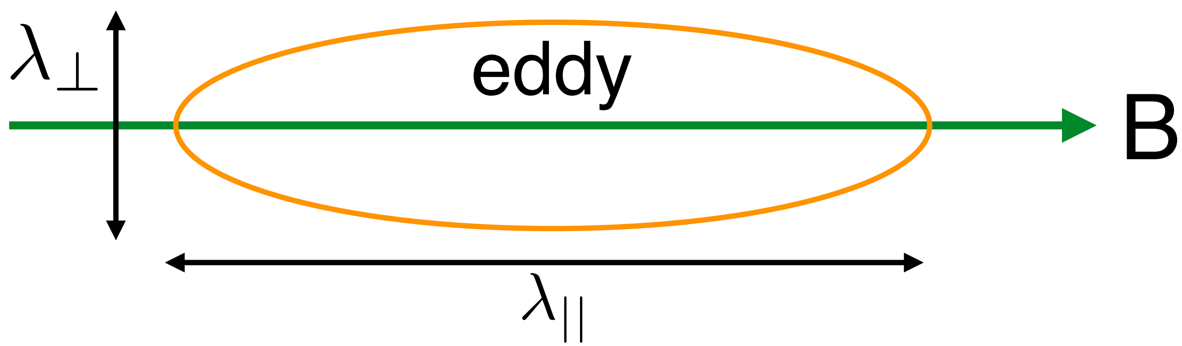
$$k_{||} \propto k_{\perp}^{2/3}$$

·Turbulent motion perpendicular to B obeys Kolmogolov law

$$E(k_{\perp}) \propto k_{\perp}^{-5/3}$$

$$\boxed{k_{||} c_A \sim k_{\perp} v_k}$$ <u>:critical balance</u>

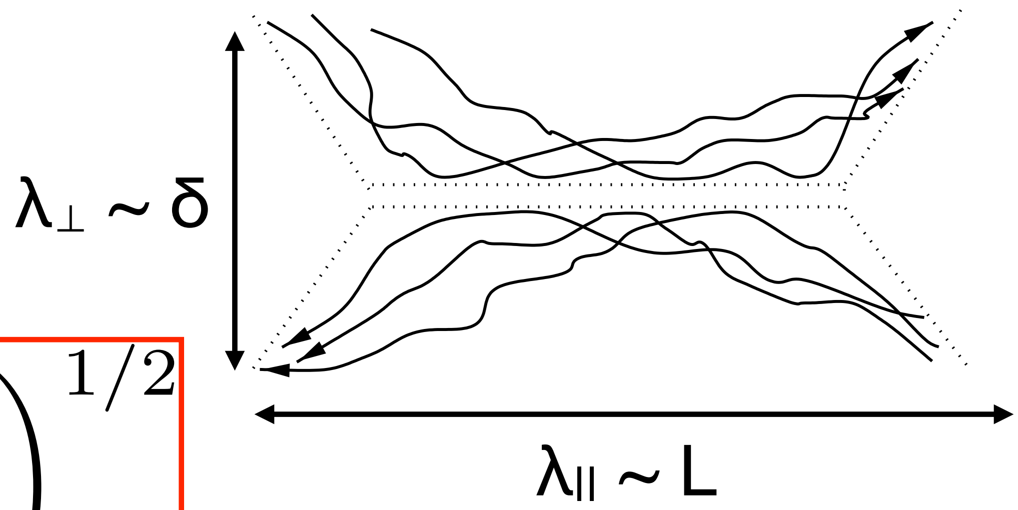
Theoretical Explanation



$$\frac{\lambda_{||}}{l} \sim \left(\frac{\lambda_\perp}{l}\right)^{2/3} \left(\frac{v_l}{c_A}\right)^{-4/3}$$: MHD turbulence

$$\frac{v_{\rm in}}{c_A} = \frac{\delta}{L} \boxed{\sim \frac{\lambda_\perp}{\lambda_{||}}}$$

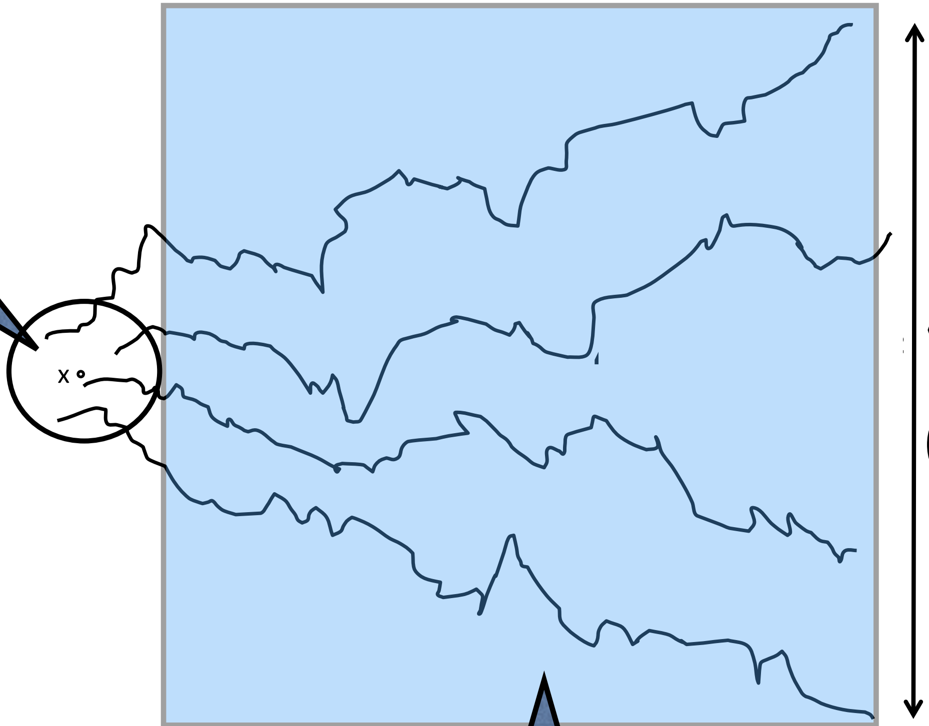
$$\sim \boxed{\left(\frac{v_l}{c_A}\right)^2 \left(\frac{l}{L}\right)^{1/2}}$$



3.1. Theoretical Explanation

ref) Eyink, Lazarian, Vishniac, (2011), ApJ, 743, 51.

initial location



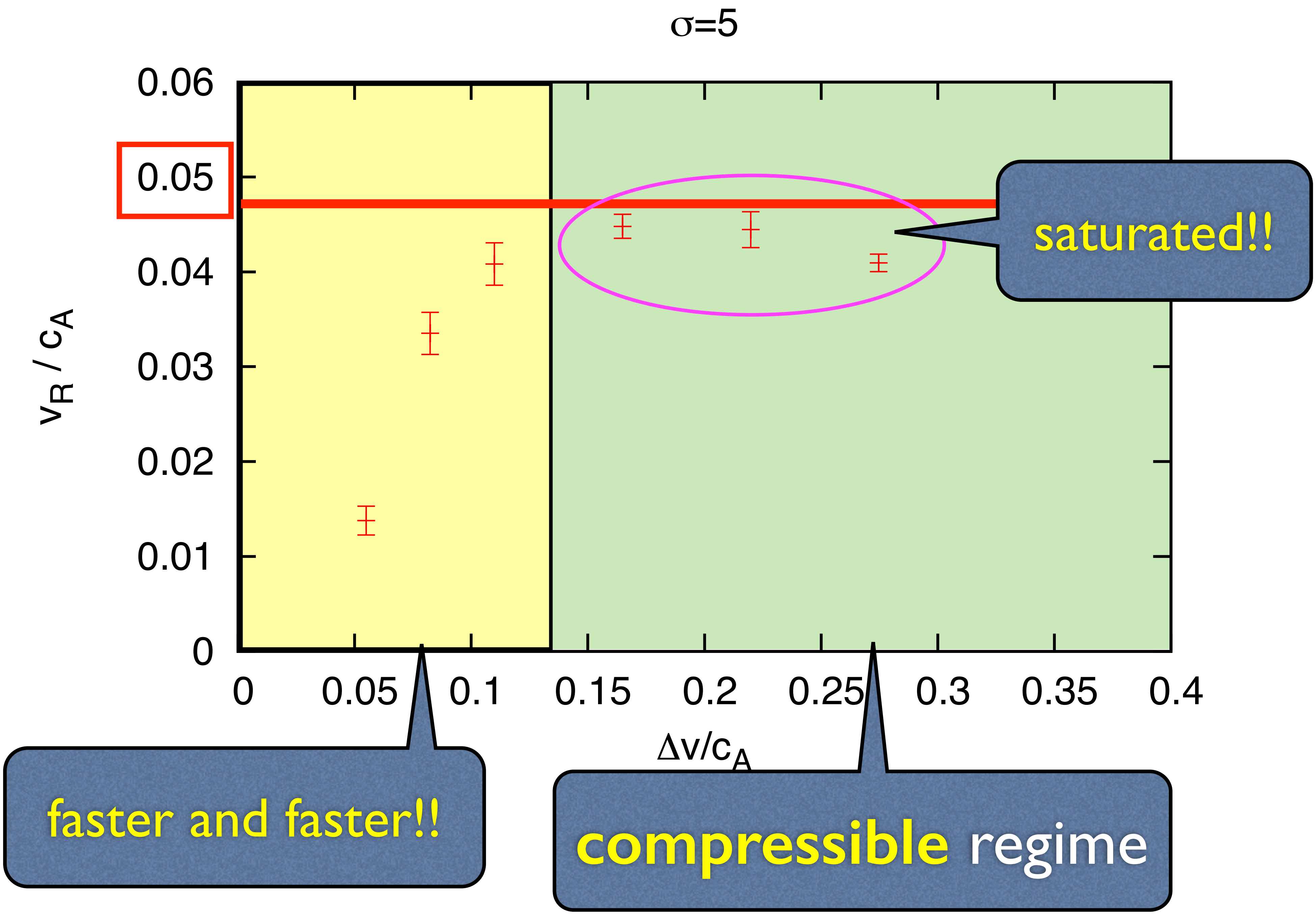
$$\langle y^2(t)\rangle \sim (L_x^3/L_i) M_A^4 (t/t_A)^3$$

Diffusion= independent of initial width

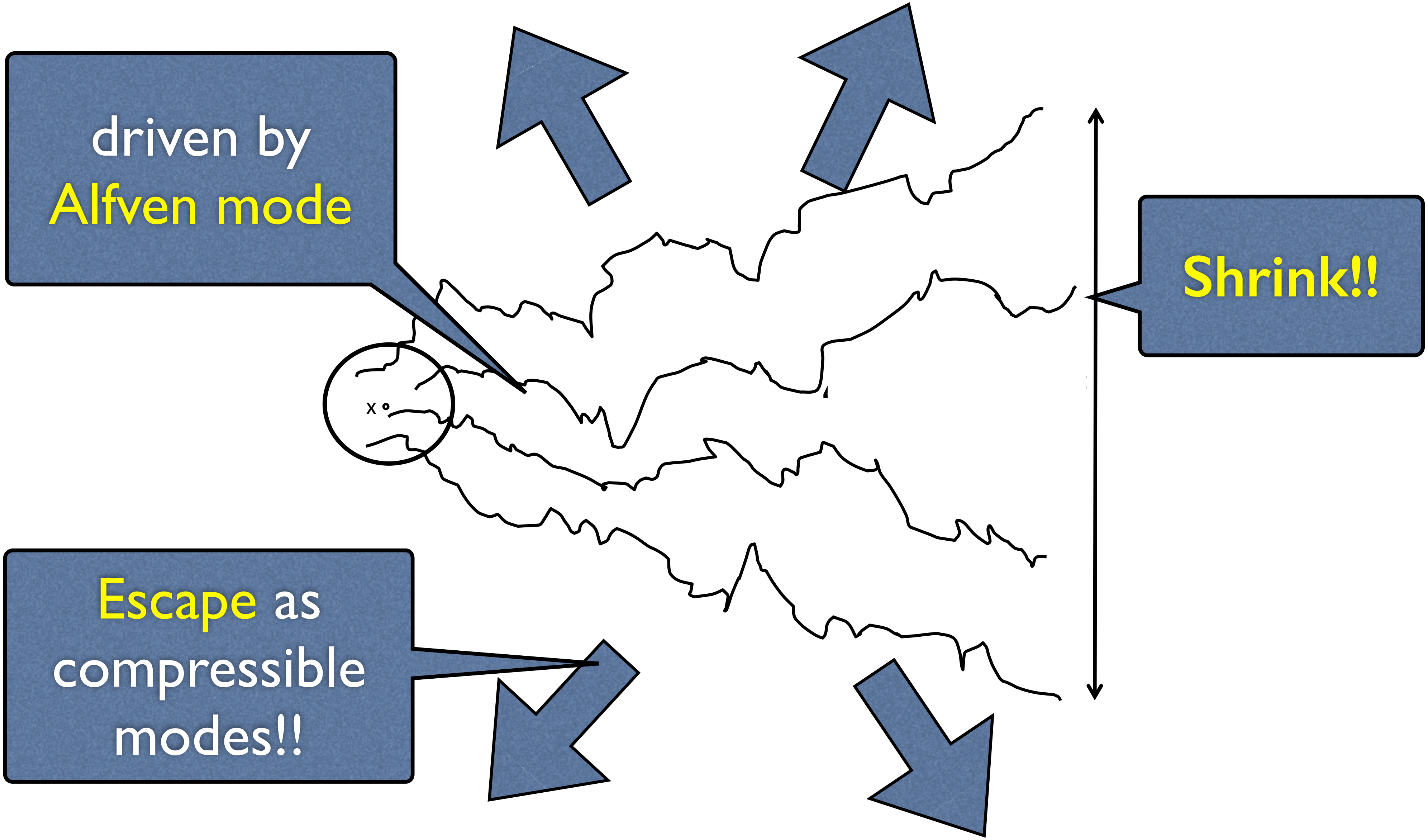
Richardson Two Particle Diffusion:D⊥

$$\frac{\delta}{L_x} = M_A^2 \min\left\{\left(\frac{L_x}{L_i}\right)^{1/2}, \left(\frac{L_i}{L_x}\right)^{1/2}\right\}$$

3.4. Turbulence-Strength Dependence



3.5. Compressible Effect



3.7. Compressible Effects

$$\boxed{\frac{v_{\rm in}}{c_A} = \frac{\rho_{\rm out}}{\rho_{\rm in}} \frac{v_{\rm out}}{c_A} \boxed{\frac{\delta}{L}}}$$

Incompressible:
(LV99)

$$\frac{\delta}{L} \simeq \min\left[\left(\frac{L}{l}\right)^{1/2}, \left(\frac{l}{L}\right)^{1/2}\right]\left(\frac{v_l}{c_A}\right)^2$$

escaping as compressible modes

compressible:

$$\frac{\delta}{L} \simeq \min\left[\left(\frac{L}{l}\right)^{1/2}, \left(\frac{l}{L}\right)^{1/2}\right]\left[\left(\frac{v_l}{c_A}\right)^2 \boxed{- C\left(\frac{v_l}{c_A}\right)^4}\right]$$

Relativistic Ideal Fluid

Basic equations of relativistic hydrodynamics (RHD):

$$\boxed{D}\rho = -\rho\boxed{\nabla_\mu}u^\mu$$:Mass Conservation

$$\rho\boxed{h}\boxed{D}u_\mu = -\boxed{\nabla_\mu}p$$: Equation of Motion

$$\rho\boxed{D}e = -p\boxed{\nabla_\mu}u^\mu$$: Equation of Energy

$$\gamma_{\mu\nu} = \eta_{\mu\nu} + u_\mu u_\nu$$: spatial projection tensor

$$\partial_\mu = \eta_{\mu\nu}\partial^\nu = (-u_\mu u_\nu + \gamma_{\mu\nu})\partial^\nu$$
$$\equiv -u_\mu D + \nabla_\mu$$:3+1 decomposition

3.6. Compressible Effect: 2

ref) Banerjee & Galtier, PRE 87, 013019, (2013).

Energy cascade law in MHD turbulence:

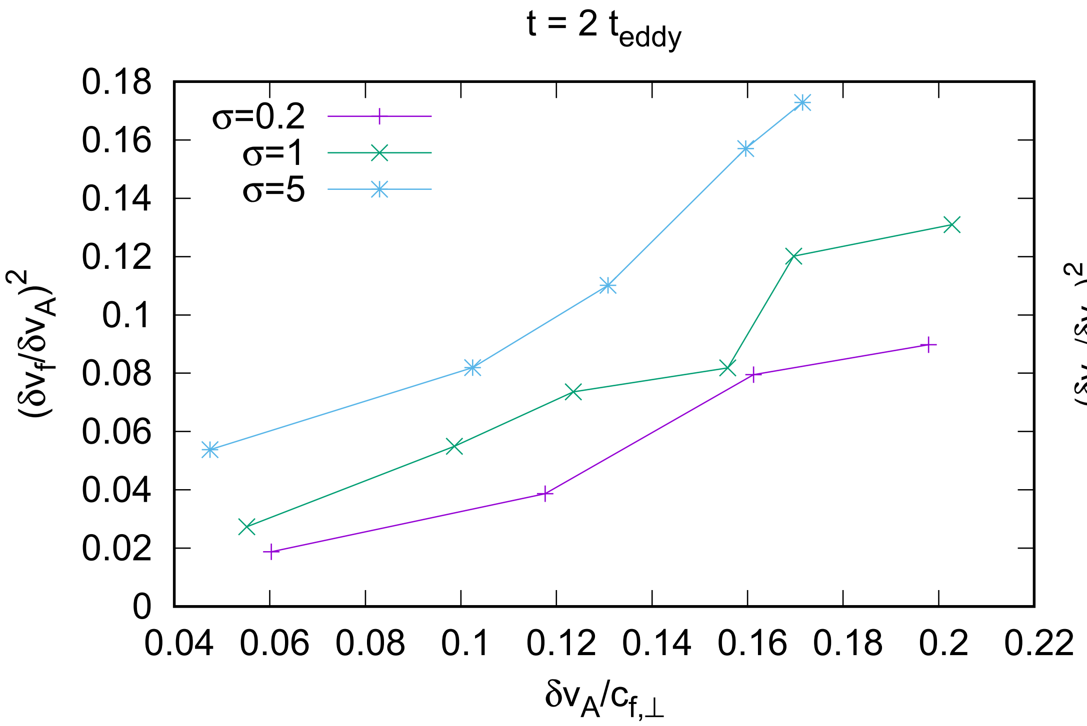
$$-4\epsilon = \nabla \cdot \mathbf{F} + \boxed{B_0^2 S}$$

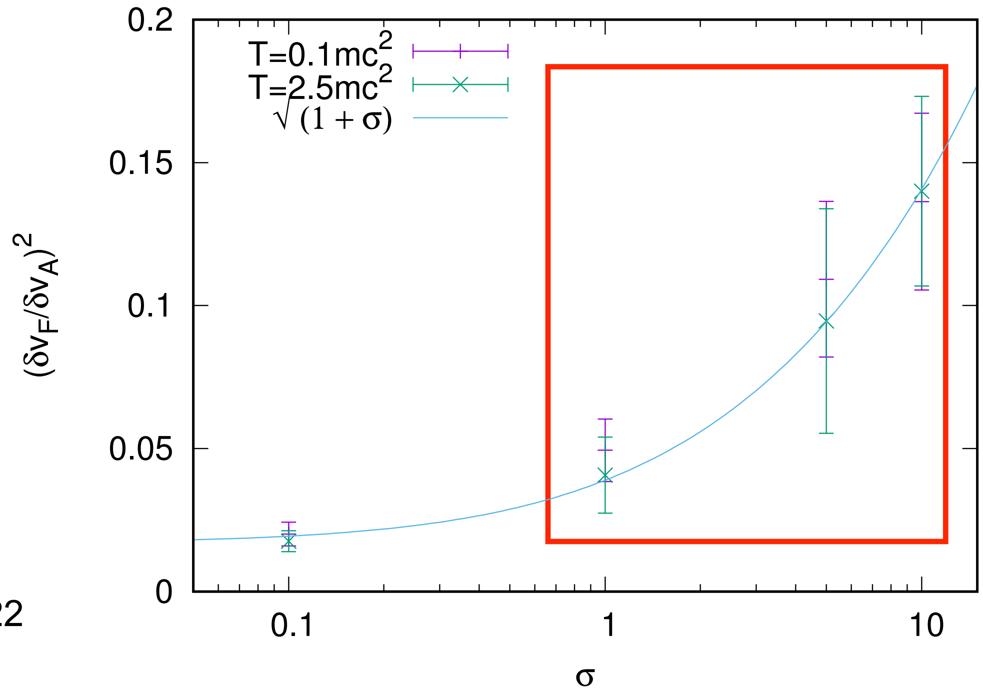
$$\boxed{\epsilon_{\rm eff} = \epsilon + B_0^2 S/4}$$

$$\boxed{\begin{aligned}
&\langle \Psi_{\mathbf{v}} \rangle = \frac{B_0^2}{2} \left\langle \delta(\boldsymbol{\nabla} \cdot \mathbf{v}) \delta\left(\frac{1}{\sqrt{\rho}}\right) \bar{\delta}(\sqrt{\rho}) - \bar{\delta}(\boldsymbol{\nabla} \cdot \mathbf{v}) \right\rangle, \\
&\langle \Psi_{\mathbf{v_A}} \rangle \\
&\quad = \mathbf{B_0} \cdot \left\langle \boldsymbol{\nabla} \left(\frac{1}{\sqrt{\rho}}\right) \left[(\mathbf{B_0} \cdot \mathbf{v}') \left\{ \rho' \delta\left(\frac{1}{\sqrt{\rho}}\right) \right\} - (\mathbf{B_0} \cdot \mathbf{v}) \frac{\delta\rho}{2\sqrt{\rho'}} \right] \right. \\
&\qquad \left. - \boldsymbol{\nabla}' \left(\frac{1}{\sqrt{\rho'}}\right) \left[(\mathbf{B_0} \cdot \mathbf{v}) \left\{ \rho \delta\left(\frac{1}{\sqrt{\rho}}\right) \right\} - (\mathbf{B_0} \cdot \mathbf{v}') \frac{\delta\rho}{2\sqrt{\rho}} \right] \right\rangle.
\end{aligned}}$$

3.6. Alfven to Fast Mode Convergion

ref) MT & Lazarian (2016), ApJL, 831, 11.





fast mode power linearly increases with Alfven component

compression mode increases with σ-parameter

$$
\begin{aligned}
(\delta V)_{\rm f}^2/(\delta V)_{\rm A}^2 &\propto (\delta V)_{\rm A}/c_{{\rm fast},\perp} && (\text{when } \sigma \ll 1), \\
&\propto \boxed{(1+\sigma)^{1/2}}(\delta V)_{\rm A}/c_{{\rm fast},\perp} && (\text{when } \sigma \gtrsim 1).
\end{aligned}
$$

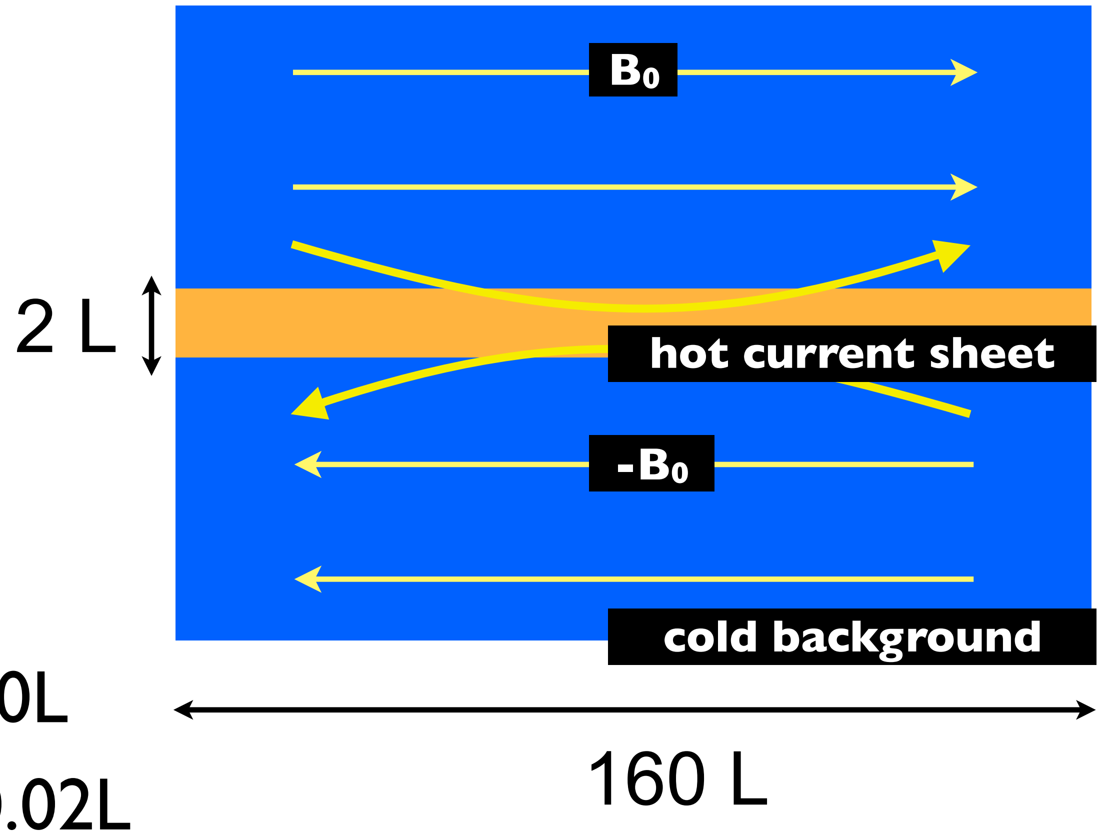
3.8. initial condition for self-generated turbulence

Initial condition:

- Harris current sheet
- cold upstream flow

  ($T \sim 0.1 mc^2$)
- hot current sheet

  ($T_{sheet} \sim mc^2$)
- Box size:



- mesh size: 160L×20L×40L

$\Delta_x, \Delta_y, \Delta_z, \sim 0.04L, 0.02L, 0.02L$

- uniform resistivity
- Large Lundquist number:

  $S_L \sim 10^5$
- Poynting dominated :

  $\sigma = 5$

$$\boxed{\sigma \equiv \frac{[\mathbf{E} \times \mathbf{B} c/4\pi]}{\rho h c^2 \gamma^2 v}}$$

2. Poynting Dominated Plasma of Astrophysical Phenomena