CANS1D モジュール hdroe

流体エンジン Roe＋MUSCL＋TVD法

ver. 0.2

1 はじめに

このモジュールは、流体力学方程式・MHD 方程式を Roe ＋ MUSCL ＋ TVD 法で解くためのものです。

2 基礎方程式

以下で $\gamma=$ 定数 は比熱比、他の記号は通常の意味。

2.1 サブルーチン roe_a；移流

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho V_x)=0 \tag{1}$$

2.2 サブルーチン roe_h；流体

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho V_x)=0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho V_x)+\frac{\partial}{\partial x}\left[(\rho V_x^2+p)\right]=0 \tag{3}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1}+\frac{1}{2}\rho V_x^2\right)\right]+\frac{\partial}{\partial x}\left[\left(\frac{\gamma}{\gamma-1}p+\frac{1}{2}\rho V_x^2\right)V_x\right]=0 \tag{4}$$

$$p=\frac{k_{\mathrm{B}}}{m}\rho T \tag{5}$$

2.3 サブルーチン roe_h_g；流体重力

$$\frac{\partial}{\partial t}(\rho)+\frac{\partial}{\partial x}(\rho V_x)=0 \tag{6}$$

$$\frac{\partial}{\partial t}(\rho V_x)+\frac{\partial}{\partial x}\left[(\rho V_x^2+p)\right]=\rho g \tag{7}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1}+\frac{1}{2}\rho V_x^2\right)\right]+\frac{\partial}{\partial x}\left[\left(\frac{\gamma}{\gamma-1}p+\frac{1}{2}\rho V_x^2\right)V_x\right]=\rho g V_x \tag{8}$$

$$p=\frac{k_{\mathrm{B}}}{m}\rho T \tag{9}$$

2.4 サブルーチン roe_h_c；流体非一様断面

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{10}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p)S\right] = p\frac{dS}{dx} \tag{11}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma - 1} + \frac{1}{2}\rho V_x^2\right)S\right] + \frac{\partial}{\partial x}\left[\left(\frac{\gamma}{\gamma - 1}p + \frac{1}{2}\rho V_x^2\right)V_x S\right] = 0 \tag{12}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{13}$$

S は断面積、

2.5 サブルーチン roe_h_cg；流体非一様断面重力

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{14}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p)S\right] = \rho g S + p\frac{dS}{dx} \tag{15}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma - 1} + \frac{1}{2}\rho V_x^2\right)S\right] + \frac{\partial}{\partial x}\left[\left(\frac{\gamma}{\gamma - 1}p + \frac{1}{2}\rho V_x^2\right)V_x S\right] = (\rho g V_x)S \tag{16}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{17}$$

2.6 サブルーチン roe_ht；等温流体

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) = 0 \tag{18}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p)\right] = 0 \tag{19}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{20}$$

温度 T は既知の定数。

2.7 サブルーチン roe_ht_g；等温流体重力

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) = 0 \tag{21}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p)\right] = \rho g \tag{22}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{23}$$

温度 T は既知の定数。

2.8 サブルーチン roe_ht_c；等温流体非一様断面

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{24}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p)S\right] = p\frac{dS}{dx} \tag{25}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{26}$$

S は断面積、温度 T は既知の定数。

2.9 サブルーチン roe_ht_cg；等温流体非一様断面重力

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{27}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p)S\right] = \rho g S + p\frac{dS}{dx} \tag{28}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{29}$$

温度 T は既知の定数。

2.10 サブルーチン roe_m；MHD

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) = 0 \tag{30}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})\right] = 0 \tag{31}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})\right] = 0 \tag{32}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}\left(E_z\right) = 0 \tag{33}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right)\right] + \frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi}\right)\right] = 0 \tag{34}$$

$$E_z = -V_x B_y + V_y B_x \tag{35}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{36}$$

$$B^2 = B_x^2 + B_y^2, \quad V^2 = V_x^2 + V_y^2 \tag{37}$$

磁場 B_x は既知の定数。

2.11 サブルーチン roe_m_g；MHD 重力

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) = 0 \tag{38}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})\right] = \rho g_x \tag{39}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})\right] = \rho g_y \tag{40}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}(E_z) = 0 \tag{41}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right)\right] + \frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi}\right)\right] = \rho(g_x V_x + g_y V_y) \tag{42}$$

$$E_z = -V_x B_y + V_y B_x \tag{43}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{44}$$

$$B^2 = B_x^2 + B_y^2, \quad V^2 = V_x^2 + V_y^2 \tag{45}$$

磁場 B_x は既知の定数。

2.12 サブルーチン roe_m_c；MHD 非一様断面

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{46}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})S\right] = (p + \frac{B_y^2}{8\pi})\frac{dS}{dx} \tag{47}$$

$$\frac{\partial}{\partial t}(\rho V_y S) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})S\right] = 0 \tag{48}$$

$$\frac{\partial}{\partial t}(B_y S) - \frac{\partial}{\partial x}(E_z S) = 0 \tag{49}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right)S\right] + \frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi}\right)S\right] = 0 \tag{50}$$

$$E_z = -V_x B_y + V_y B_x \tag{51}$$

$$B^2 = B_x^2 + B_y^2, \quad V^2 = V_x^2 + V_y^2 \tag{52}$$

S は断面積、

2.13 サブルーチン roe_m_cg；MHD 非一様断面重力

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{53}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})S\right] = \rho g_x S + (p + \frac{B_y^2}{8\pi})\frac{dS}{dx} \tag{54}$$

$$\frac{\partial}{\partial t}(\rho V_y S) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})S\right] = \rho g_y S \tag{55}$$

$$\frac{\partial}{\partial t}(B_y S) - \frac{\partial}{\partial x}(E_z S) = 0 \tag{56}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right)S\right] + \frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi}\right)S\right] = \rho(g_x V_x + g_y V_y)S \tag{57}$$

$$E_z = -V_x B_y + V_y B_x \tag{58}$$

$$B^2 = B_x^2 + B_y^2, \quad V^2 = V_x^2 + V_y^2 \tag{59}$$

2.14 サブルーチン roe_m_cgr；MHD 非一様断面重力回転

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{60}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})S\right] = \rho\left[g_x + (V_y^2 - \frac{B_y^2}{4\pi\rho})\frac{1}{R}\frac{dR}{dx}\right]S + (p + \frac{B_y^2}{8\pi})\frac{dS}{dx} \tag{61}$$

$$\frac{\partial}{\partial t}(\rho V_y R S) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})RS\right] = 0 \tag{62}$$

$$\frac{\partial}{\partial t}\left(\frac{B_y S}{R}\right) - \frac{\partial}{\partial x}\left(\frac{E_z S}{R}\right) = 0 \tag{63}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right)S\right] + \frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi}\right)S\right] = \rho g_x V_x S \tag{64}$$

$$E_z = -V_x B_y + V_y B_x \tag{65}$$

$$B^2 = B_x^2 + B_y^2, \quad V^2 = V_x^2 + V_y^2 \tag{66}$$

$R(x)$ は回転軸からの距離

2.15 サブルーチン roe_m3；3 成分 MHD

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) = 0 \tag{67}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})\right] = 0 \tag{68}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})\right] = 0 \tag{69}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial x}\left[(\rho V_x V_z - \frac{B_x B_z}{4\pi})\right] = 0 \tag{70}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}(E_z) = 0 \tag{71}$$

$$\frac{\partial}{\partial t}(B_z) + \frac{\partial}{\partial x}(E_y) = 0 \tag{72}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right)\right] + \frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi} + \frac{B_z E_y}{4\pi}\right)\right] = 0 \tag{73}$$

$$E_y = -V_z B_x + V_x B_z, \quad E_z = -V_x B_y + V_y B_x \tag{74}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{75}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2, \quad V^2 = V_x^2 + V_y^2 + V_z^2 \tag{76}$$

磁場 B_x は既知の定数。

2.16 サブルーチン roe_m3_g；3 成分 MHD 重力

$$\frac{\partial}{\partial t}(\rho) + \frac{\partial}{\partial x}(\rho V_x) = 0 \tag{77}$$

$$\frac{\partial}{\partial t}(\rho V_x) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})\right] = \rho g_x \tag{78}$$

$$\frac{\partial}{\partial t}(\rho V_y) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})\right] = \rho g_y \tag{79}$$

$$\frac{\partial}{\partial t}(\rho V_z) + \frac{\partial}{\partial x}\left[(\rho V_x V_z - \frac{B_x B_z}{4\pi})\right] = \rho g_z \tag{80}$$

$$\frac{\partial}{\partial t}(B_y) - \frac{\partial}{\partial x}(E_z) = 0 \tag{81}$$

$$\frac{\partial}{\partial t}(B_z) + \frac{\partial}{\partial x}(E_y) = 0 \tag{82}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right)\right] + \frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma-1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi} + \frac{B_z E_y}{4\pi}\right)\right] = \rho(g_x V_x + g_y V_y + g_z V_z) \tag{83}$$

$$E_y = -V_z B_x + V_x B_z, \quad E_z = -V_x B_y + V_y B_x \tag{84}$$

$$p = \frac{k_{\mathrm{B}}}{m}\rho T \tag{85}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2, \quad V^2 = V_x^2 + V_y^2 + V_z^2 \tag{86}$$

磁場 B_x は既知の定数。

2.17 サブルーチン roe_m3_c；3 成分 MHD 非一様断面

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{87}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})S\right] = (p + \frac{B_y^2 + B_z^2}{8\pi})\frac{dS}{dx} \tag{88}$$

$$\frac{\partial}{\partial t}(\rho V_y S) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})S\right] = 0 \tag{89}$$

$$\frac{\partial}{\partial t}(\rho V_z S) + \frac{\partial}{\partial x}\left[(\rho V_x V_z - \frac{B_x B_z}{4\pi})S\right] = 0 \tag{90}$$

$$\frac{\partial}{\partial t}(B_y S) - \frac{\partial}{\partial x}(E_z S) = 0 \tag{91}$$

$$\frac{\partial}{\partial t}(B_z S) + \frac{\partial}{\partial x}(E_y S) = 0 \tag{92}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma - 1} + \frac{1}{2}\rho V^2 + \frac{B^2}{8\pi}\right)S\right] + \frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma - 1}p + \frac{1}{2}\rho V^2)V_x - \frac{B_y E_z}{4\pi} + \frac{B_z E_y}{4\pi}\right)S\right] = 0 \tag{93}$$

$$E_y = -V_z B_x + V_x B_z, \quad E_z = -V_x B_y + V_y B_x \tag{94}$$

$$B^2 = B_x^2 + B_y^2 + B_z^2, \quad V^2 = V_x^2 + V_y^2 + V_z^2 \tag{95}$$

S は断面積、

2.18 サブルーチン roe_m3_cg；3 成分 MHD 非一様断面重力

$$\frac{\partial}{\partial t}(\rho S) + \frac{\partial}{\partial x}(\rho V_x S) = 0 \tag{96}$$

$$\frac{\partial}{\partial t}(\rho V_x S) + \frac{\partial}{\partial x}\left[(\rho V_x^2 + p + \frac{B^2}{8\pi} - \frac{B_x^2}{4\pi})S\right] = \rho g_x S + (p + \frac{B_y^2 + B_z^2}{8\pi})\frac{dS}{dx} \tag{97}$$

$$\frac{\partial}{\partial t}(\rho V_y S) + \frac{\partial}{\partial x}\left[(\rho V_x V_y - \frac{B_x B_y}{4\pi})S\right] = \rho g_y S \tag{98}$$

$$\frac{\partial}{\partial t}(\rho V_z S) + \frac{\partial}{\partial x}\left[(\rho V_x V_z - \frac{B_x B_z}{4\pi})S\right] = \rho g_z S \tag{99}$$

$$\frac{\partial}{\partial t}(B_y S) - \frac{\partial}{\partial x}(E_z S) = 0 \tag{100}$$

$$\frac{\partial}{\partial t}(B_z S) + \frac{\partial}{\partial x}(E_y S) = 0 \tag{101}$$

$$\frac{\partial}{\partial t}\left[\left(\frac{p}{\gamma-1}+\frac{1}{2}\rho V^2+\frac{B^2}{8\pi}\right)S\right]+\frac{\partial}{\partial x}\left[\left((\frac{\gamma}{\gamma-1}p+\frac{1}{2}\rho V^2)V_x-\frac{B_yE_z}{4\pi}+\frac{B_zE_y}{4\pi}\right)S\right]=\rho(g_xV_x+g_yV_y+g_zV_z)S \tag{102}$$

$$E_y=-V_zB_x+V_xB_z,\quad E_z=-V_xB_y+V_yB_x \tag{103}$$

$$B^2=B_x^2+B_y^2+B_z^2,\quad V^2=V_x^2+V_y^2+V_z^2 \tag{104}$$