Astrophysical Turbulence

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- What is turbulence?
- What has been studied about turbulence?
- What are problems involving turbulence?

What is turbulence?

Turbulence is a flow regime characterized by high momentum convection, low momentum diffusion, and pressure and velocity variation with time.

The Reynolds number characterizes whether flow conditions lead to turbulence or not.

$$\frac{\partial \vec{\upsilon}}{\partial t} = -\left(\vec{\upsilon} \bullet \vec{\nabla}\right)\vec{\upsilon} + \nu \nabla^2 \vec{\upsilon} - \frac{1}{\rho}\vec{\nabla} p$$

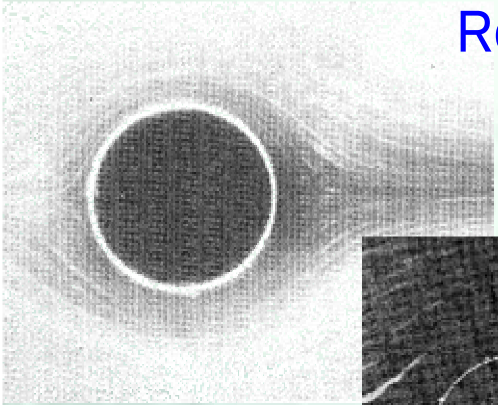
$$\downarrow \qquad\qquad \downarrow$$

$$\frac{\upsilon^2}{L} \qquad\qquad \frac{\nu\upsilon}{L^2}$$

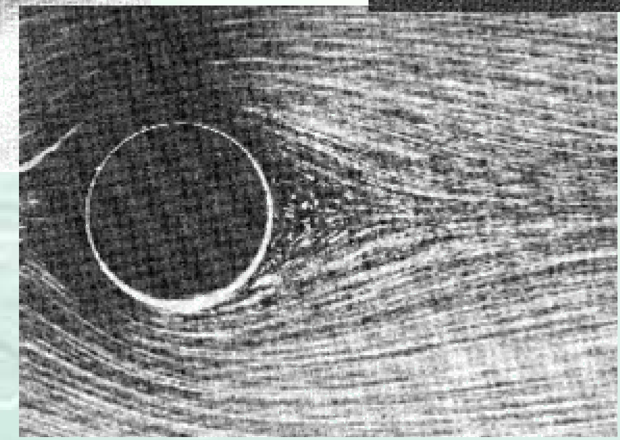
$$\mathrm{Re} \sim \frac{\upsilon^2 / L}{\nu\upsilon / L^2} \sim \frac{\upsilon L}{\nu} >\sim 100 - 1{,}000 \longrightarrow \text{turbulent!}$$

Van Dyke 1982

experiment

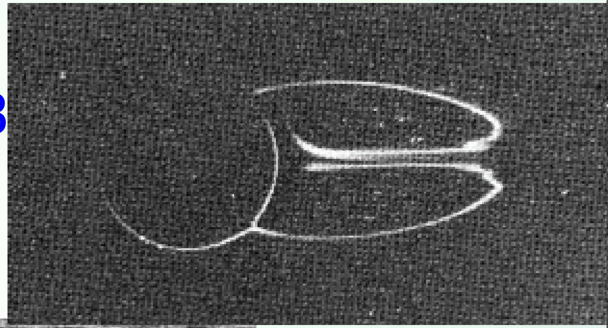


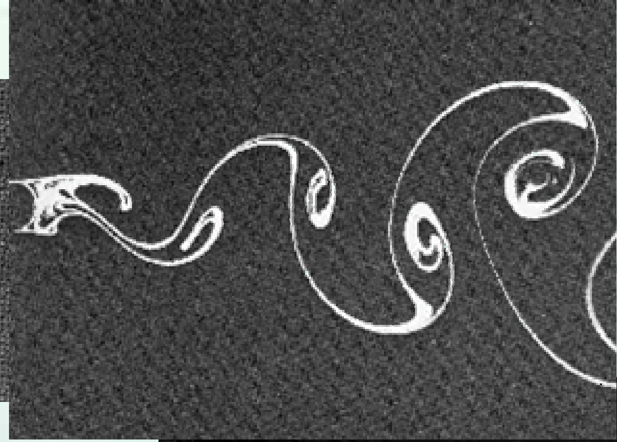
Re = 0.16



Re = 9.6

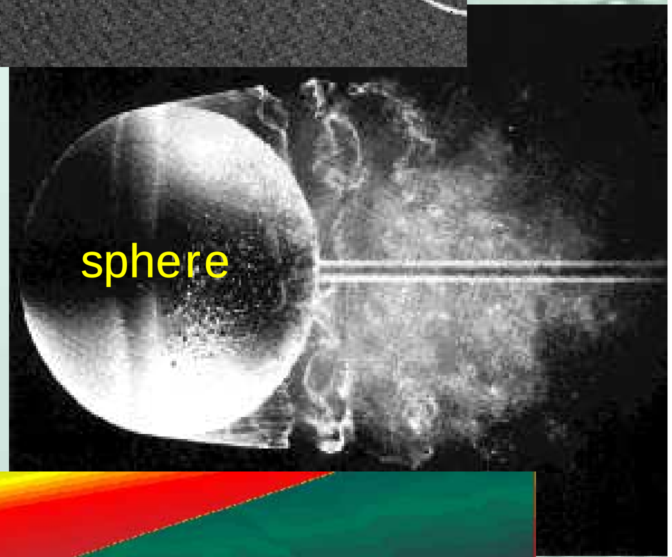
Re = 24.8





Re = 140

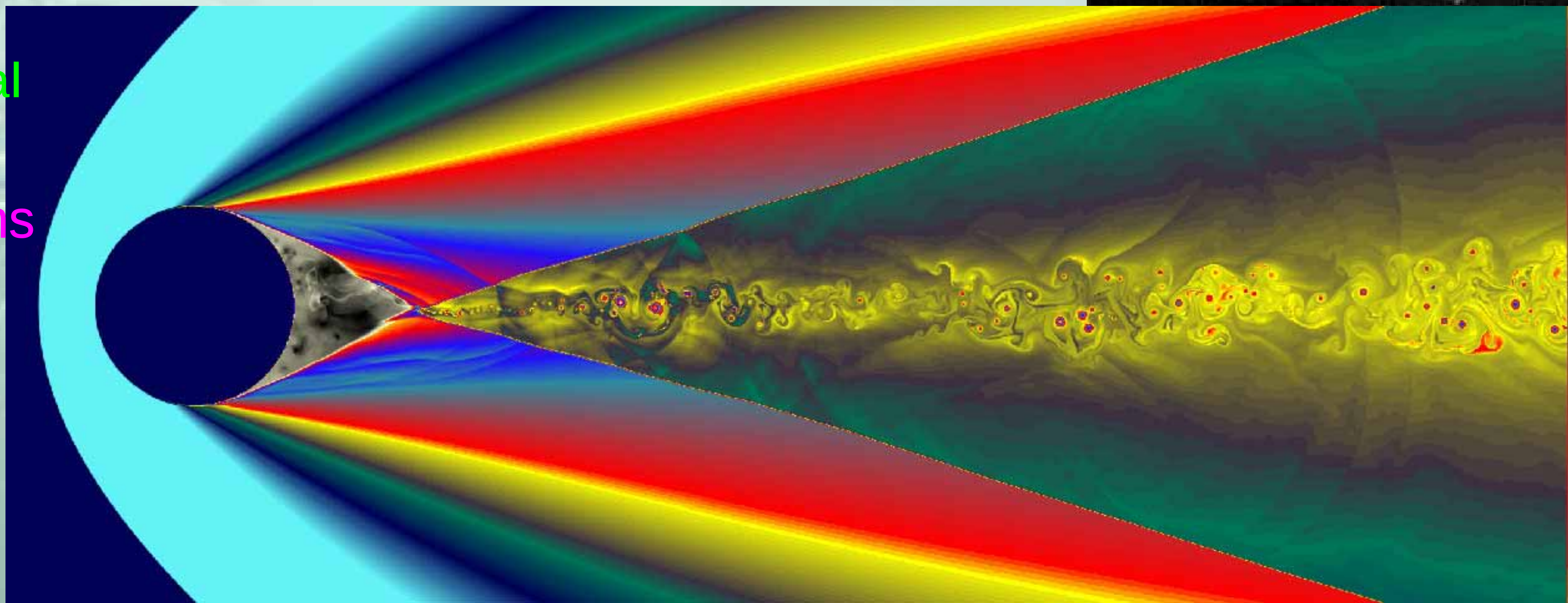
Re ~ 1,500



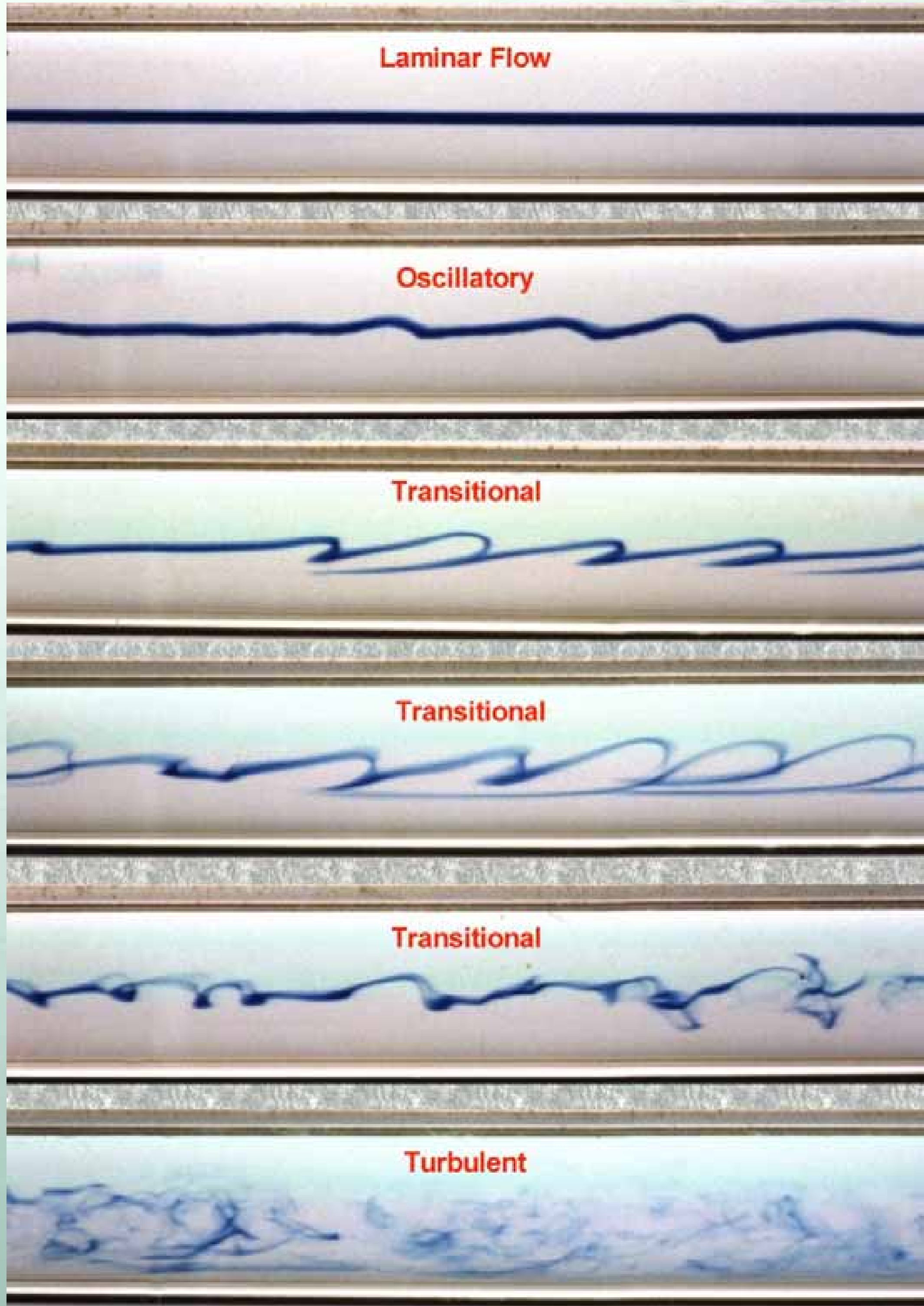
Edgar et al

numerical simulations

Re >> 1



Re

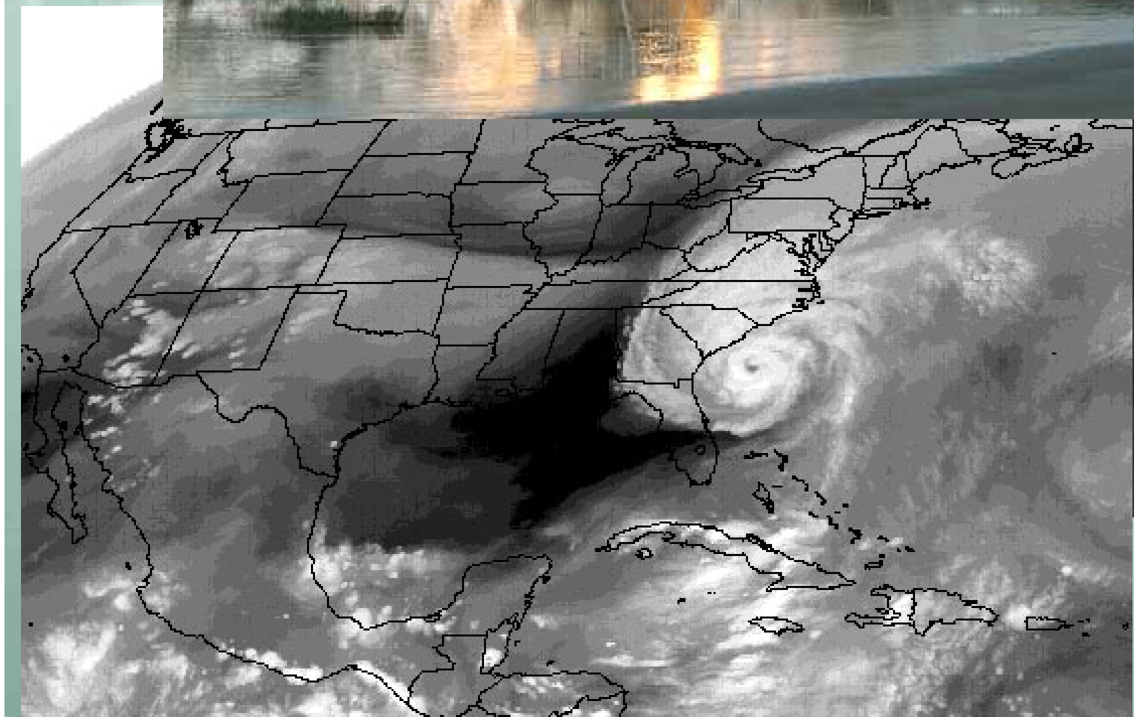

Laminar Flow
Oscillatory
Transitional
Transitional
Transitional
Turbulent

terrestrial examples 1

turbulence creating a vortex on an airplane wing

turbulent flow around an obstacle; the flow further away is laminar

terrestrial examples 2



astrophysical examples 1

Re >> 1 in astrophysical environments

a Solar filament

Jupiter's Great Red Spot from Voyager

astrophysical examples 2

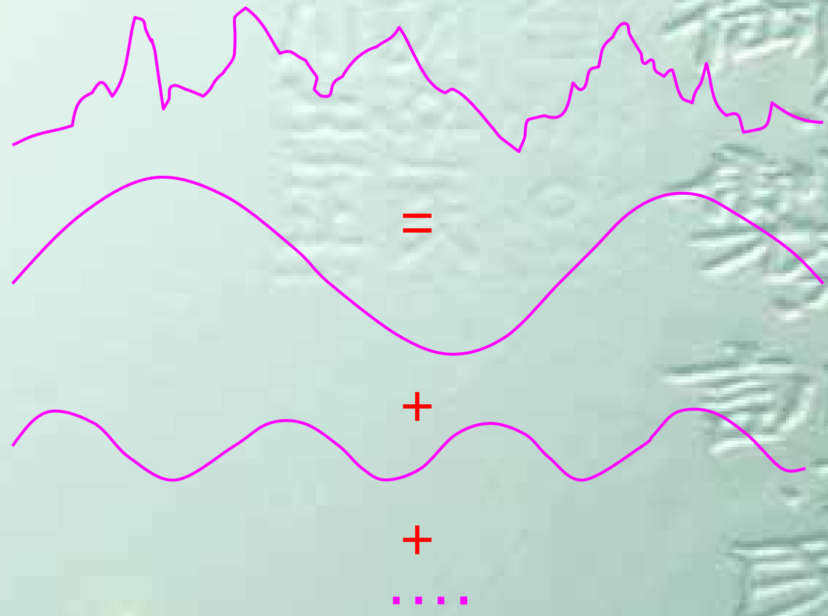
Crab Nebula – supernova remant

NGC 6302; Big, Bright, Bug Nebula – planetary nebula

Statistical description of turbulence

power spectrum, P_k - the portion of a signal's power (energy per unit wavenumber) falling within given wavenumber

$$\upsilon(\vec{r}), \rho(\vec{r}), \sqrt{\rho(\vec{r})}\upsilon(\vec{r}), B(\vec{r}), \ldots$$

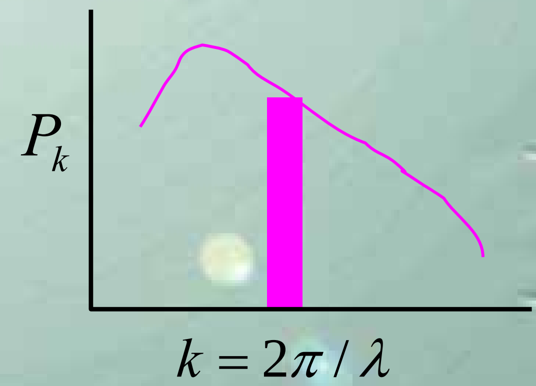


$$\longrightarrow \quad q(\vec{k}) \sim \int q(\vec{r}) d^3 r$$

Fourier transformation

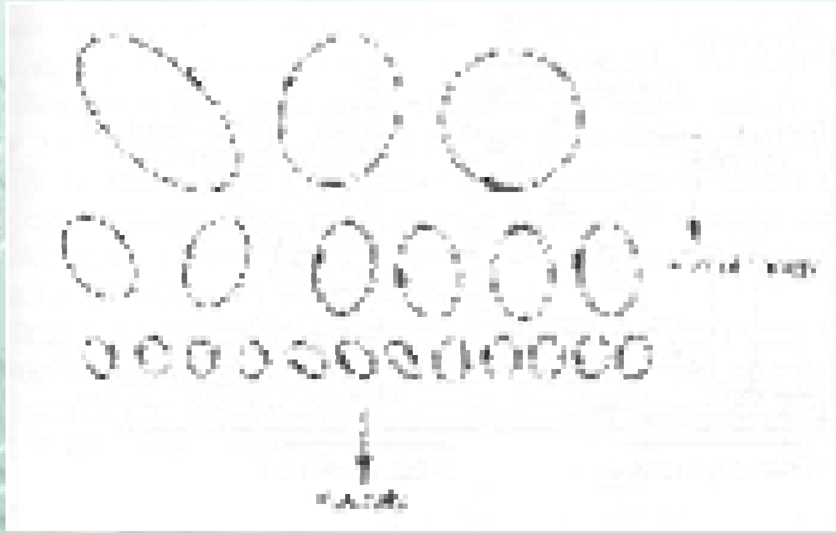
$$\longrightarrow \quad P_k \sim |\, q(\vec{k})\,|^2 \; k^2$$

$$\longleftarrow \quad \int P_k dk \sim \langle q(\vec{r}) \rangle^2$$



Theory of turbulence

Kolmogorov's theory for incompressible hydrodynamic turbulence:
it is based on the notion that that large eddies can feed energy to the smaller eddies and these in turn feed still smaller eddies, resulting in a cascade of energy from the largest eddies to the smallest ones.



a schematic representation of the turbulent energy cascade from large to small scales

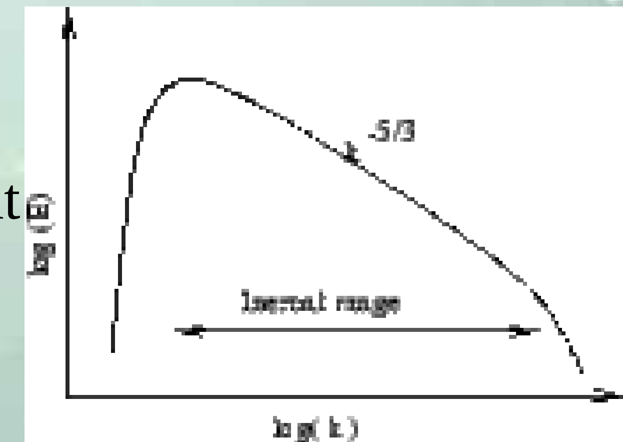
on dimensional grounds, the only way of writing ε (energy transfer rate) in terms of *V* (velocity) and *l* (sclae) is

$$\varepsilon \sim \frac{V^2}{t} \sim \frac{V^3}{l} \sim \text{constant}$$

$$V \sim l^{1/3}$$

$$P_k \sim k^{-5/3}$$

power spectrum of velocity!



the spectrum of Kolmogorov turbulence

In astrophysical environments

$$\mathrm{Re} \sim \frac{\upsilon L}{\nu} >> 1$$

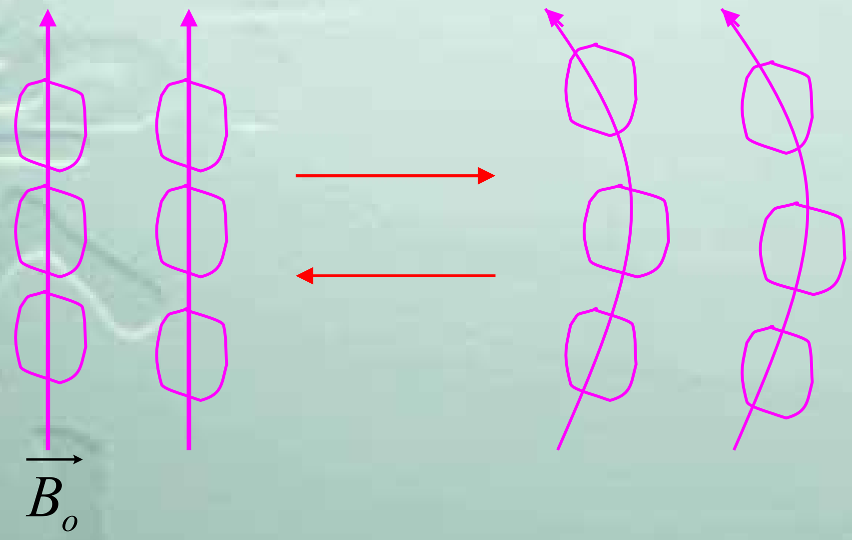
magnetic field exists in astrophysical environments:

with magnetic field

fluid → drags magnetic field

magnetic field → exerts tension and pressure

⟹ fluid and magnetic field moves together (“frozen”)



$$\frac{\partial \vec{\upsilon}}{\partial t} + \left(\vec{\upsilon} \bullet \vec{\nabla}\right)\vec{\upsilon} + \frac{1}{\rho}\vec{\nabla} p = \underline{\frac{1}{4\pi\rho}\left(\vec{\nabla} \times \vec{B}\right) \times \vec{B}}$$

$$\frac{\partial \vec{B}}{\partial t} = \underline{-\vec{\nabla} \times \left(\vec{B} \times \vec{\upsilon}\right)}$$

Super-Alfvenic turbulence

with weak regular field

(B_o small or

$$\upsilon_A = \frac{B_o}{\sqrt{4\pi\rho}} << \upsilon$$)

and still incompressible

or subsonic

⟹ super-Alfvenic turbulence

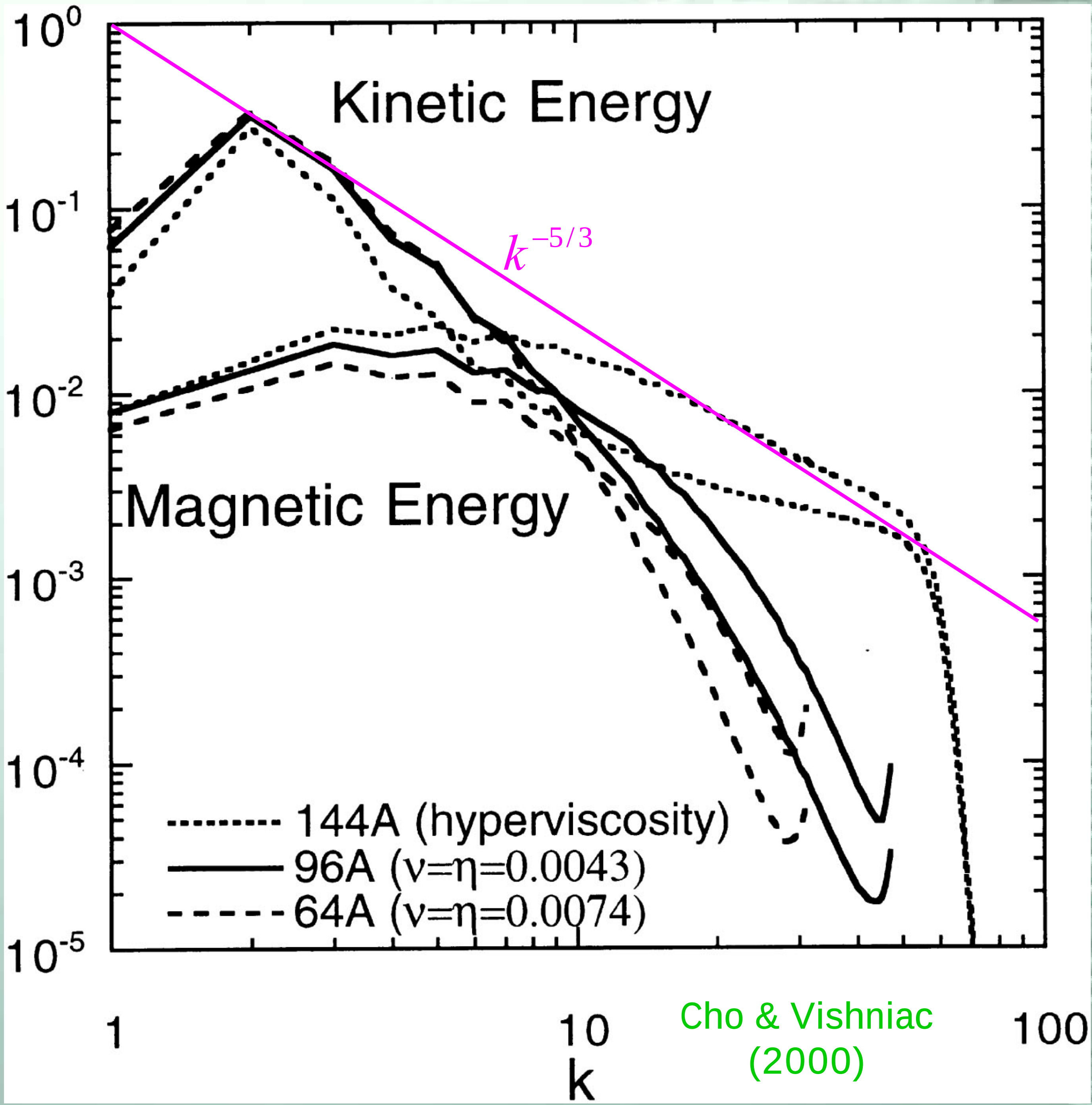
$$\left(M_A = \frac{\upsilon}{\upsilon_A} >> 1 \right)$$

more kinetic energy on larger scales, more magnetic energy on smaller scales, but

$$P_k \propto k^{-5/3}$$

for kinetic + magnetic power spectrum

(Haugen, Brandenburg et al, ...)



Cho & Vishniac (2000)

Goldreich & Sridhar model

for strong regular field

(B_o large or $\upsilon_A = \dfrac{B_o}{\sqrt{4\pi\rho}} \sim \upsilon$) ⟶ Applicable to most part of the ISM

but still incompressible or subsonic

Goldreich & Sridhar (1995) considered dynamics of Alfvenic wave packets.

Goldreich & Sridhar model

critical balance

$$\frac{l_\perp}{l_\parallel} \sim \frac{b_l}{B_o}$$

constant energy cascade

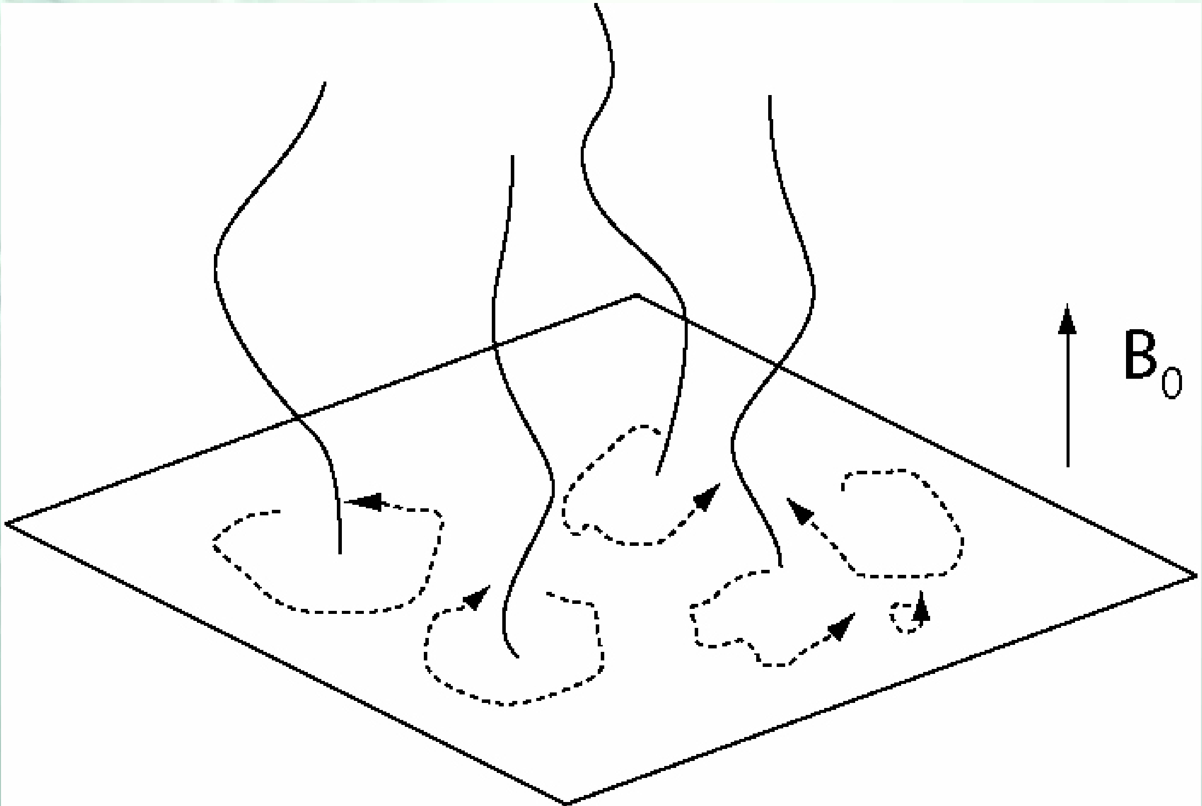
$$\varepsilon_{\text{cascade}} = \frac{b_l^2}{b_l / l_\perp} = \text{constant}$$

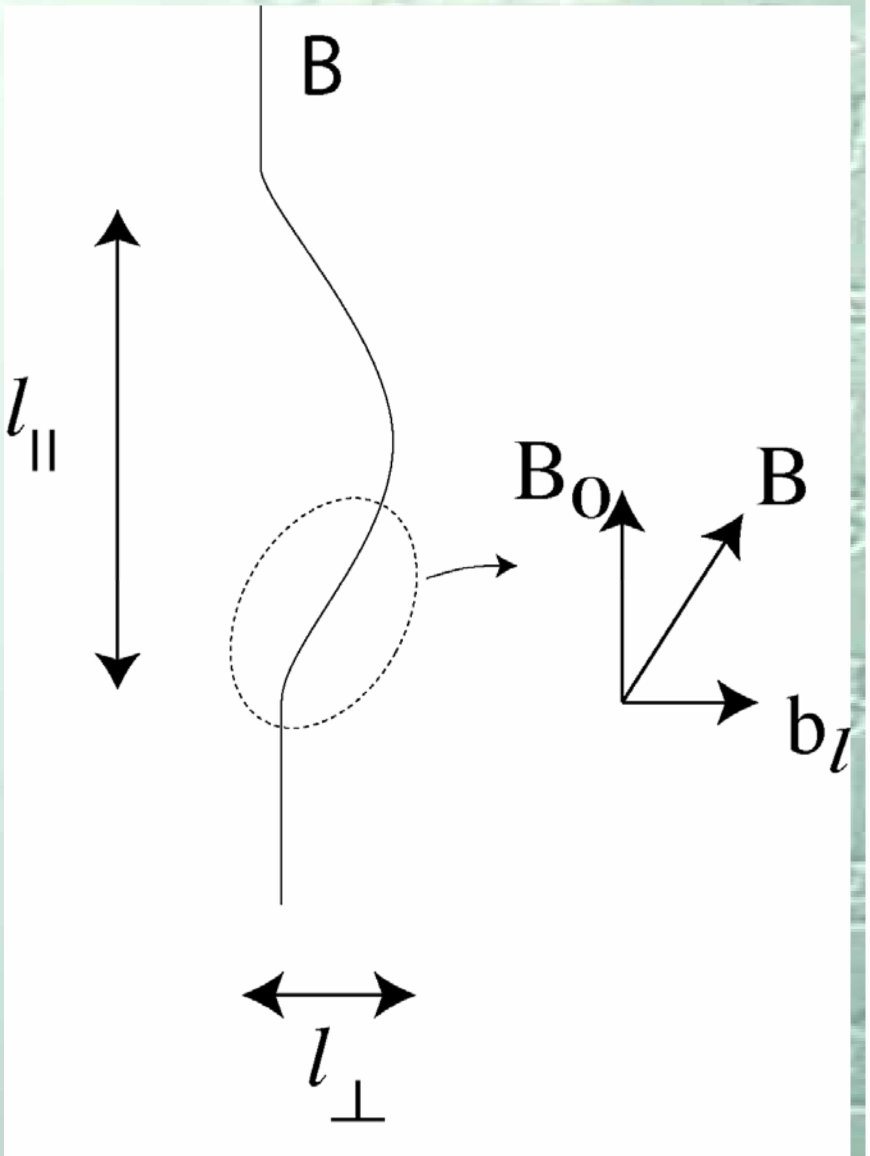
$\rightarrow$

$b \sim l_\perp^{1/3}$ or $P_k \sim k^{-5/3}$

$l_\parallel \sim l_\perp^{2/3}$

what the Goldreich & Sridhar model says





$$\upsilon_\perp \sim l_\perp^{1/3}$$

Kolmogorov

$$l_{||} \sim l_\perp^{2/3}$$

larger anisotropy at smaller scales

but astrophysical turbulence is highly compressible!

$$\frac{\delta\rho}{\rho} >> 1$$

and often highly supersonic!

$$M_s = \frac{\upsilon}{c_s} >> 1$$

so astrophysical turbulence has to be studied numerically!

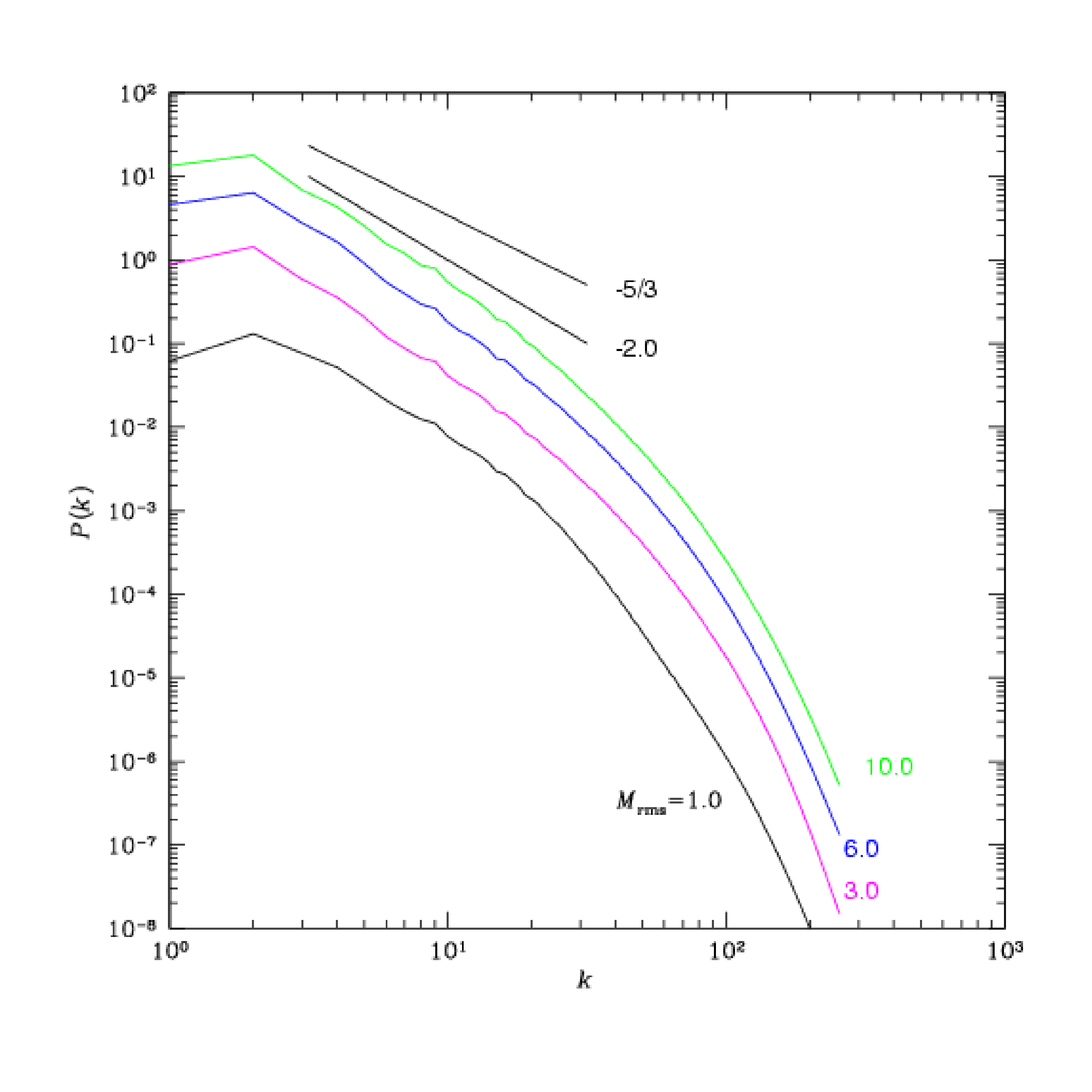
compressible hydrodynamic turbulence

sound mode (compressible mode)
    sound waves or shock waves
+ advection (incompressible or solenoid mode)
    mixing

hydrodynamics with the isothermal TVD code
3-D with 512^3 and 256^3grid zones for various M_s

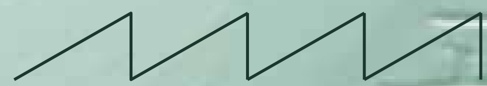
(Kim & Ryu 2005)

velocity power spectrum from 3D hydro simulations



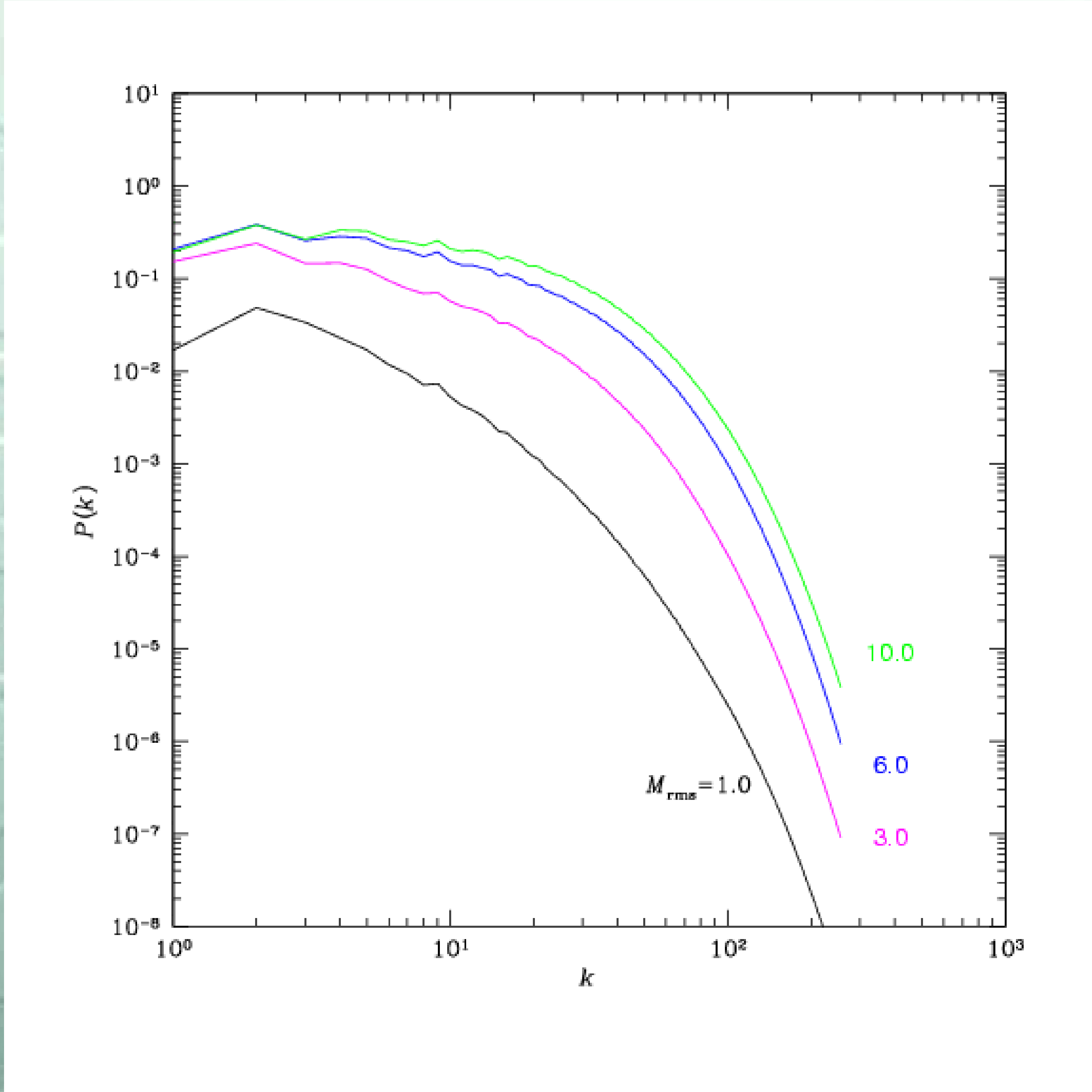
in 3D, there are both compressible and solenoidal modes

slope changes from -5/3 to -2 as M_s increases



$$P_k \sim k^{-2}$$

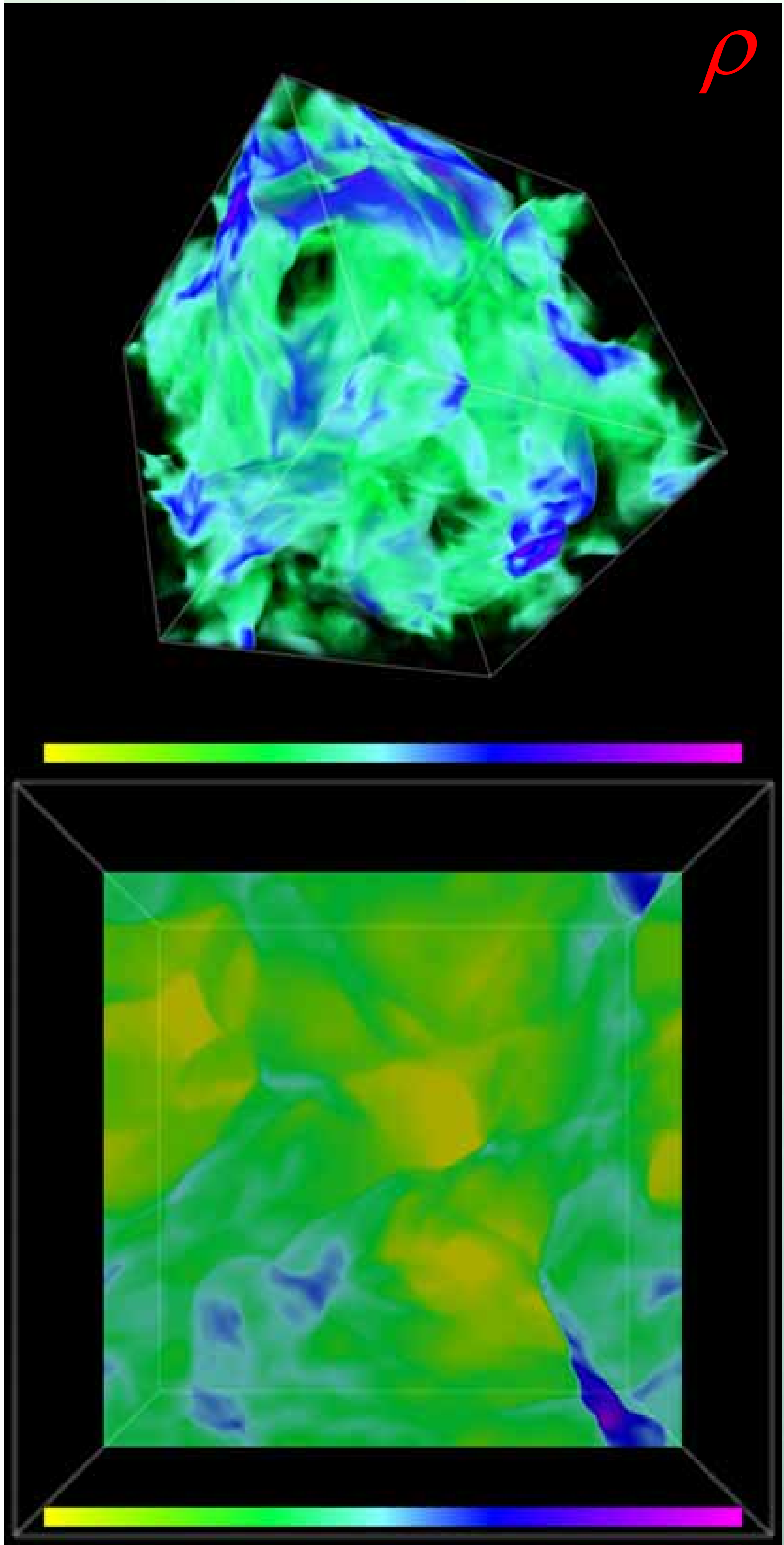
density power spectrum from 3D hydro simulations

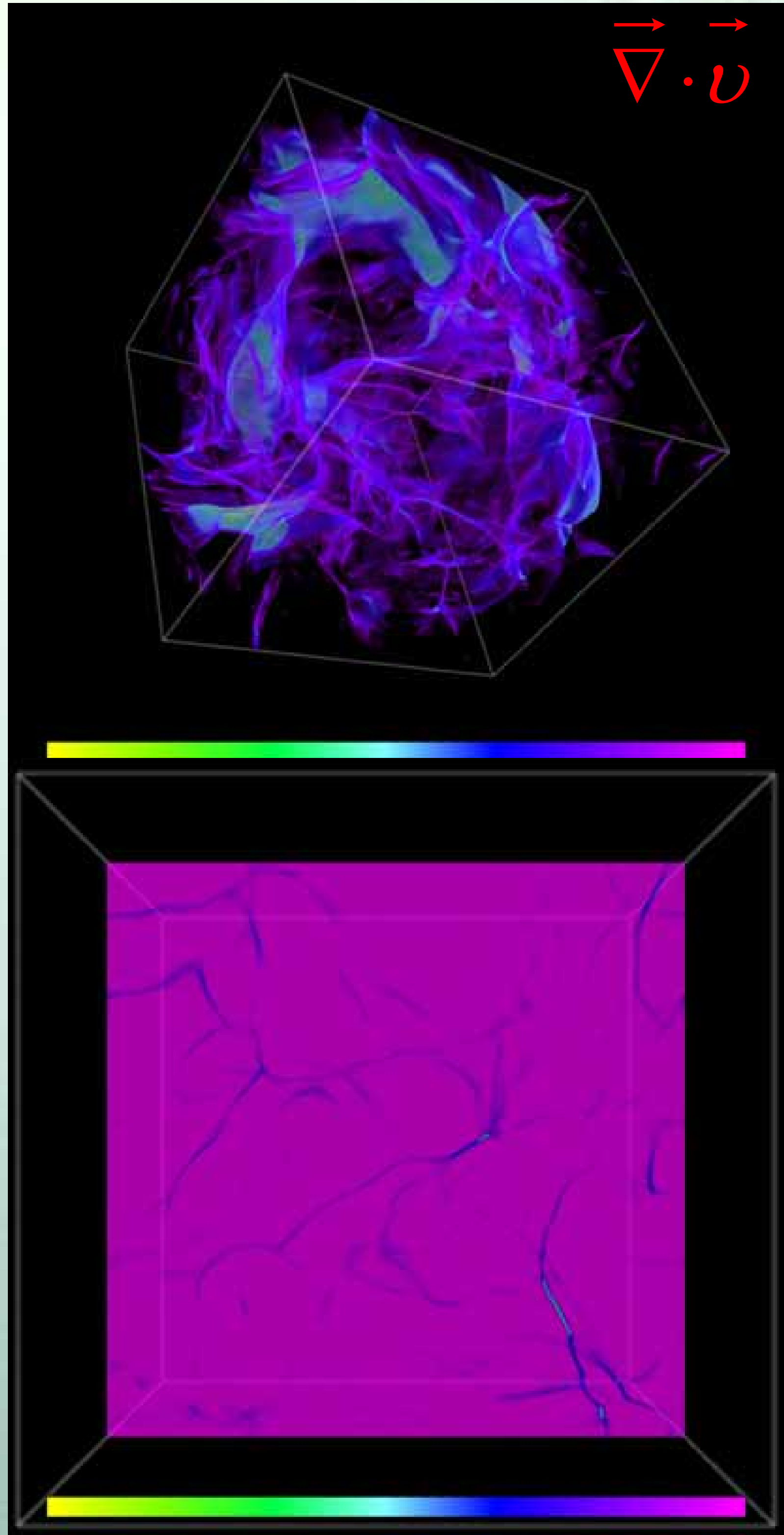


in 3D, there are both compressible and solenoidal modes

slope changes from -5/3 to 0 as M_s increases

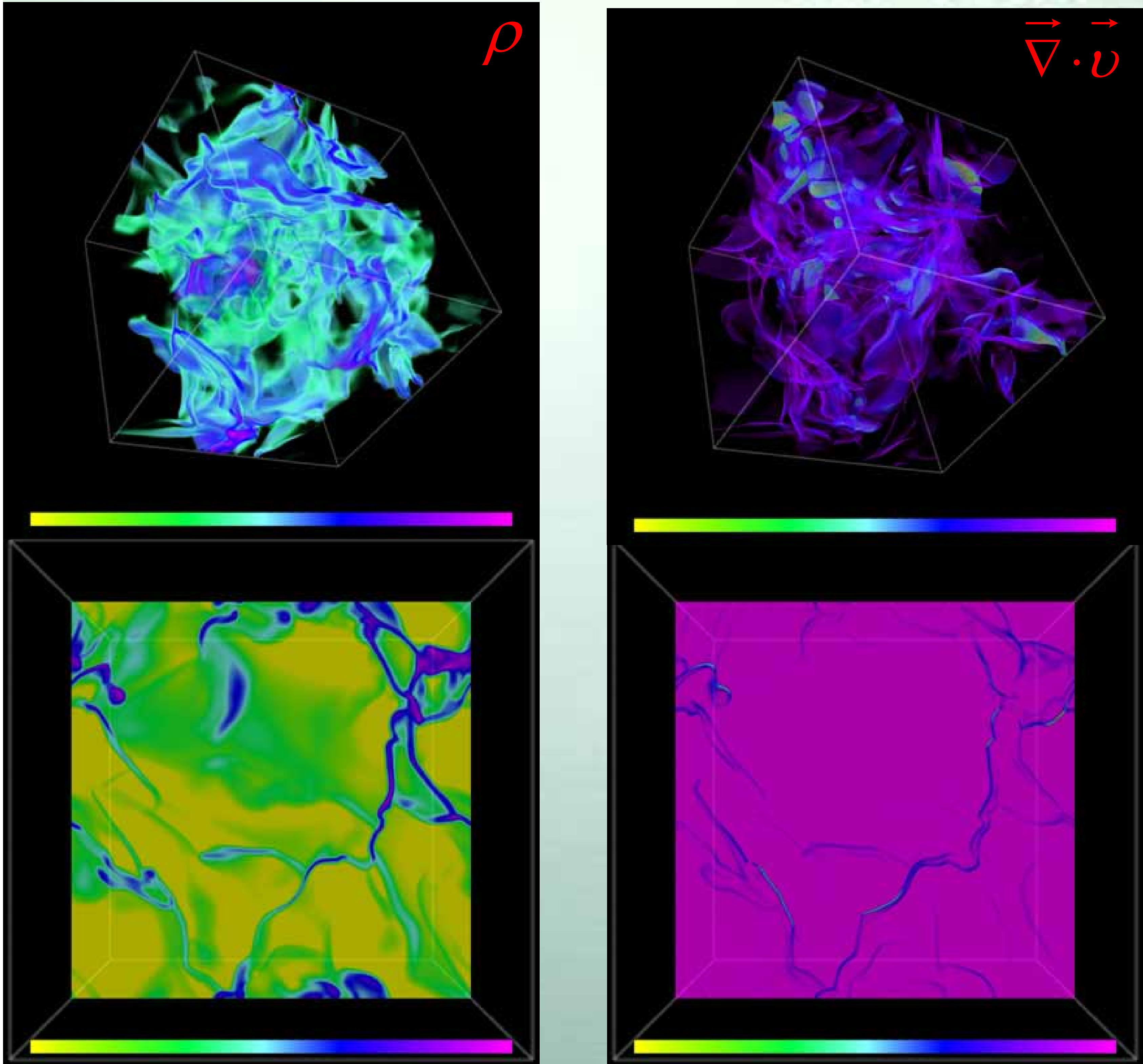
$P_k \sim k^0$





3d hydro turbulence with M_s = 1.2

“saw-toothed” distributions



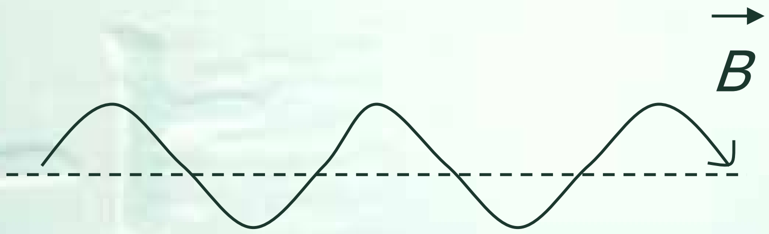
3d hydro turbulence with M_s = 12

"peaked" distribution for density

"saw-toothed" distribution for velocity

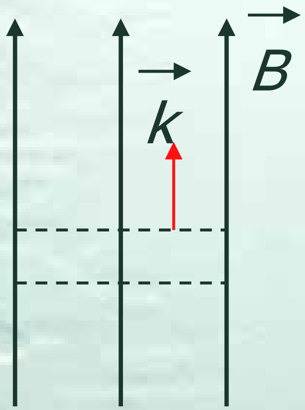
compressible magnetohydrodynamic (MHD) turbulence

MHD modes



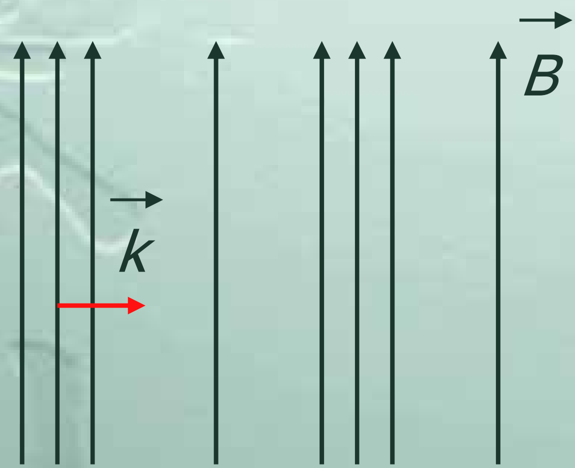
Alfven mode ($\upsilon=\upsilon_A \cos\theta$)

incompressible,
restoring force=mag. tension



slow mode ($\upsilon \sim c_s$)

for magnetically dominated plasma ($\upsilon_A \gg c_s$), this is a sound wave along magnetic field; compression of gas



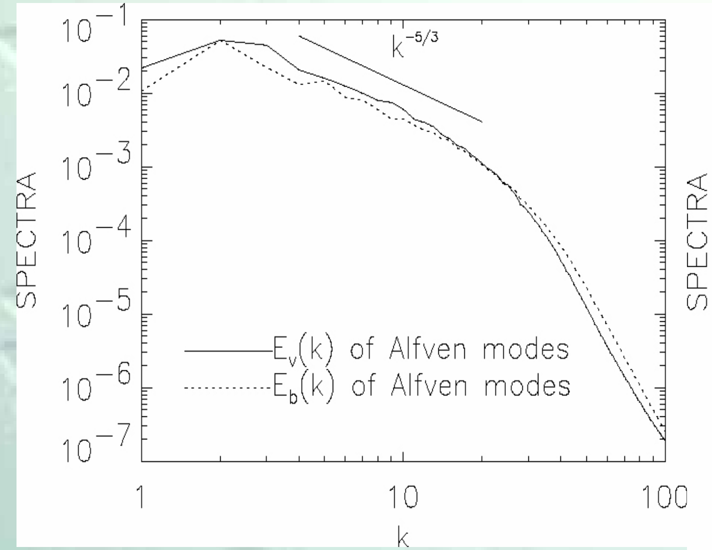
fast mode ($\upsilon \sim \upsilon_A$)

for magnetically dominated plasma ($\upsilon_A \gg c_s$), this is magnetic field compression wave; compression of B field

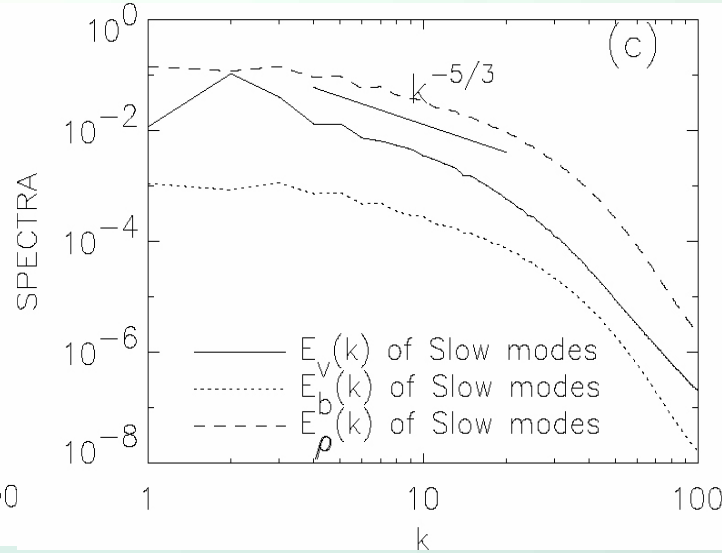
scaling relation for low β and high M_s turbulence

$$\beta = \frac{p_{\text{gas}}}{p_{\text{magnetic}}}$$

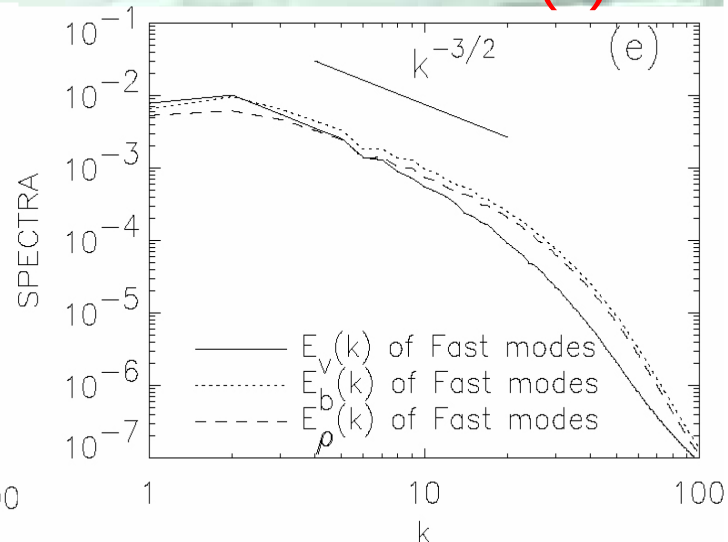
Alfven $\sim k^{-5/3}$



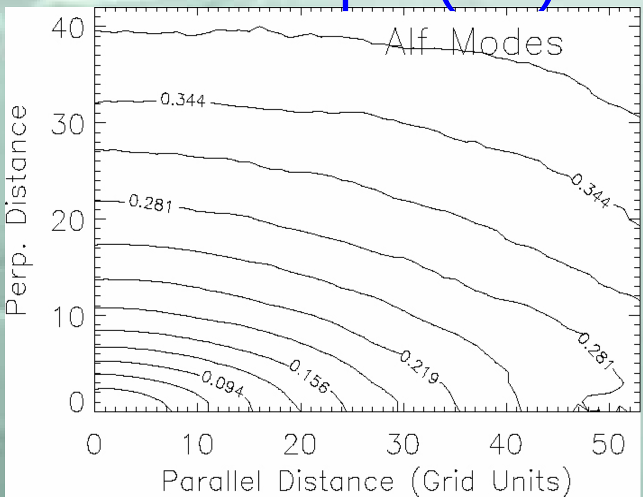
slow $\sim k^{-5/3}$



fast $\sim k^{-3/2}$(?)



anisotropic (GS)



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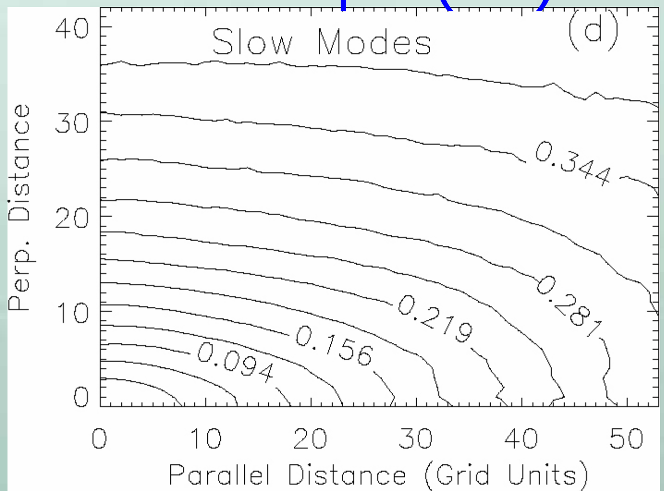
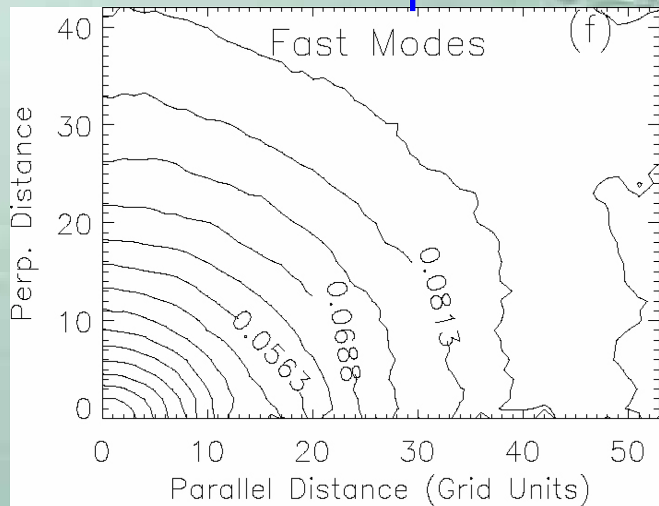


isotropic



(Cho & Lazarian 2002)

what seems to have learned about compressible turbulence with low β and high M_s (applied to the ISM)

power spectra of velocity and magnetic field

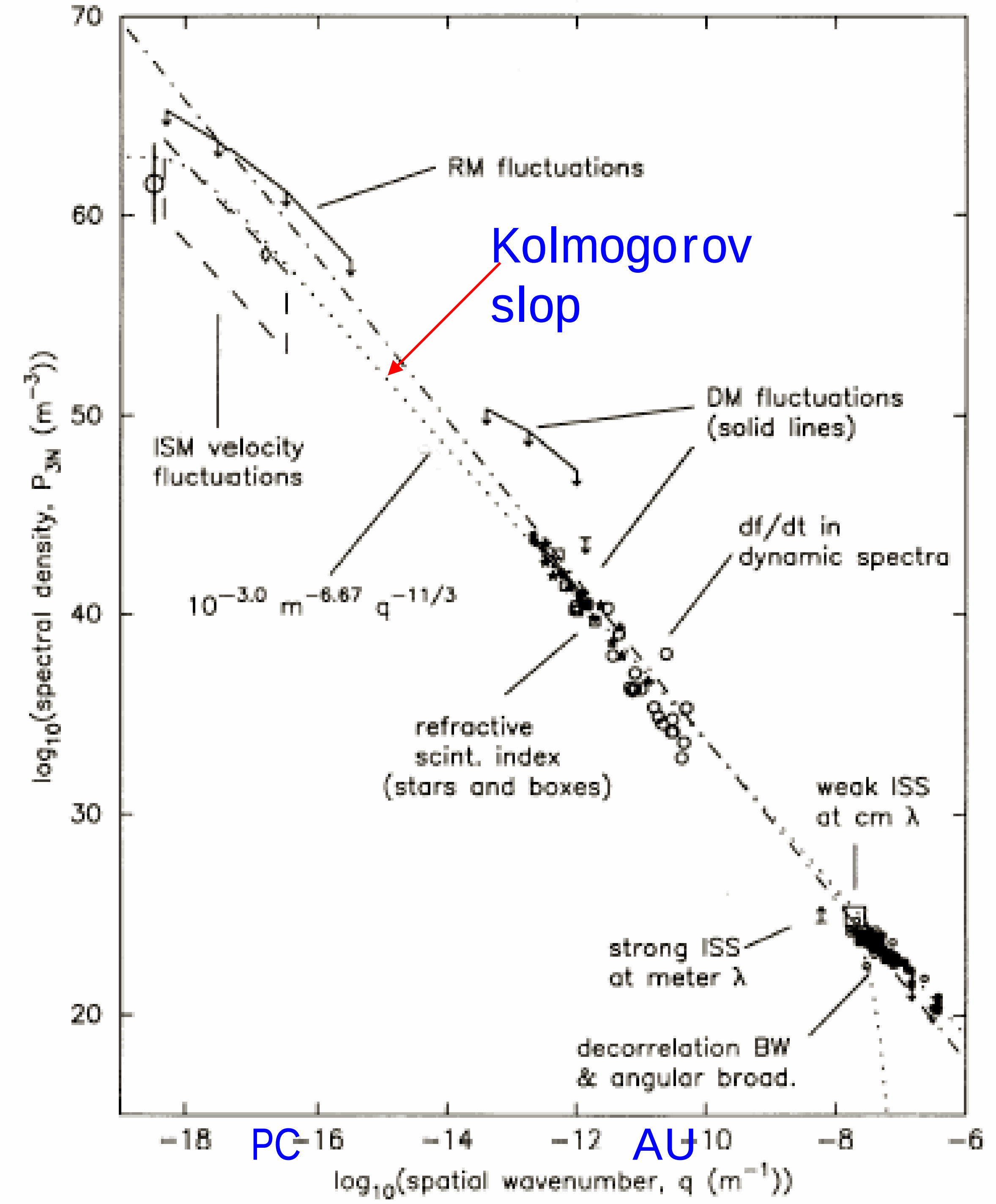
- Alfven mode: Kolmogorov slop, anisotropic – G-S model
- slow mode: Kolmogorov slop, anisotropic (passively) – G-S model
- fast mode: -1.5 slop (?), isotrpic

velocity power spectrum

- Alfven mode > fast+slow mode
- (solenoid mode > compressible mode)

density power spectrum

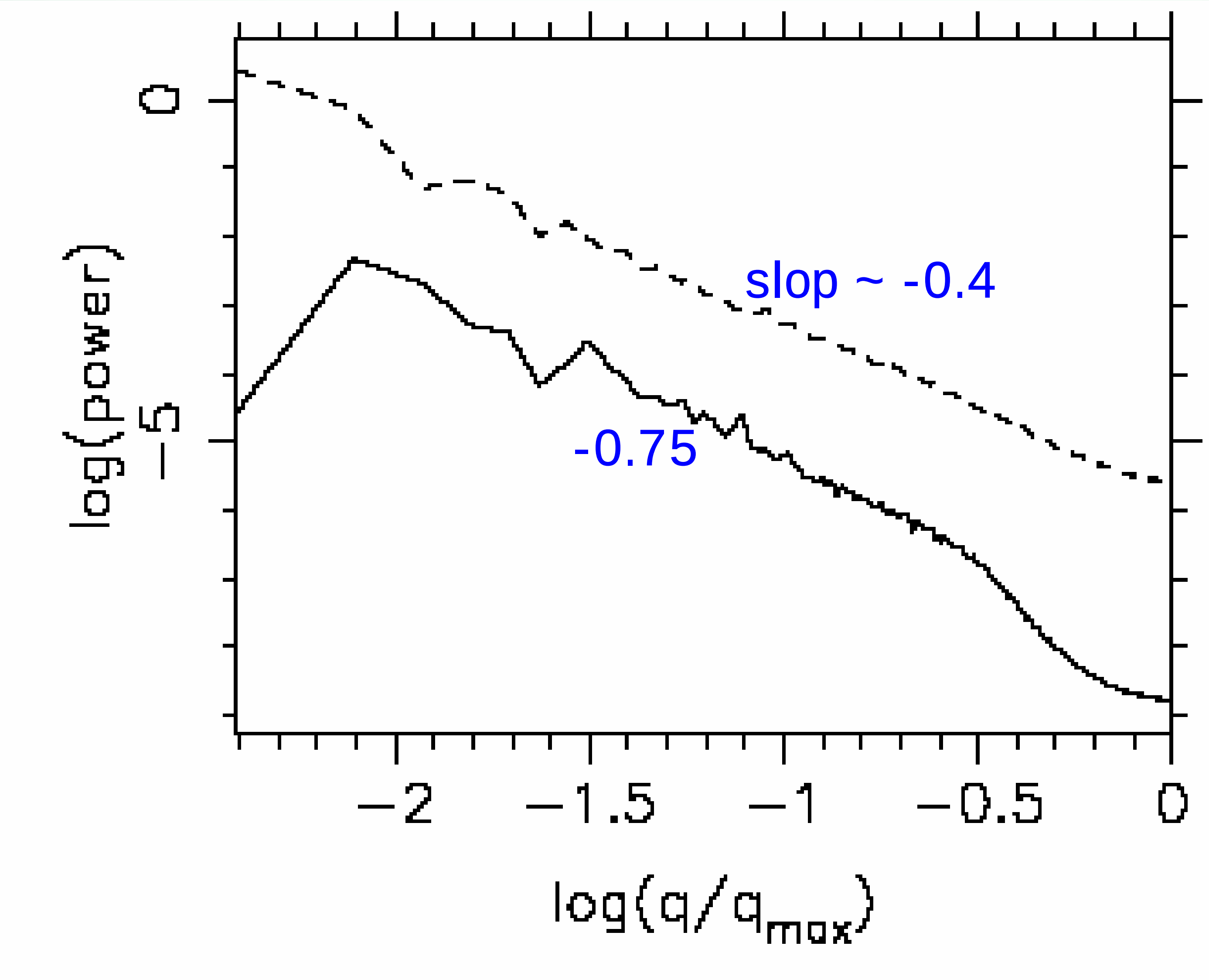
- shallower slop



power spectrum of electron column density in the interstellar medium

composite power specrum from observations of various observations

Armstrong & Spangler (1995)



density power spectrum of cold HI gas (M_s~2-3)

dash line represents a dirty PS obtained after averaging the PW of 11 channels.

solid line represents a true PS obtained after cleaning.

much shallower poswer spectrum!

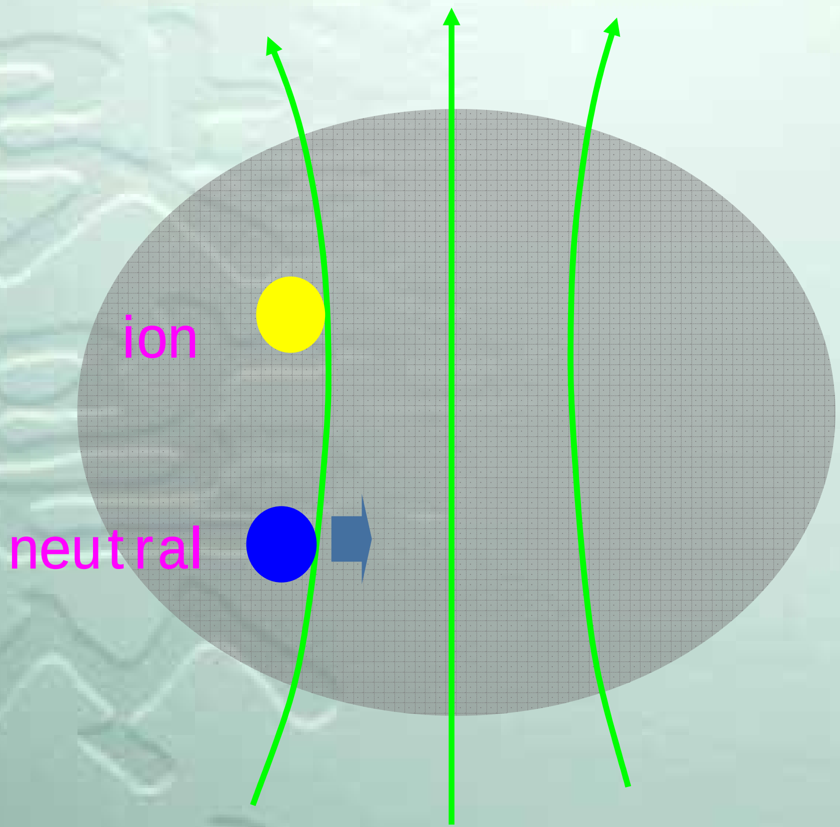
(Deshpande et al. 2000)

power spectra of various observed quantities:
seem to be compatible with that of Kolmogorov turbulence in most observations, but not in all observations

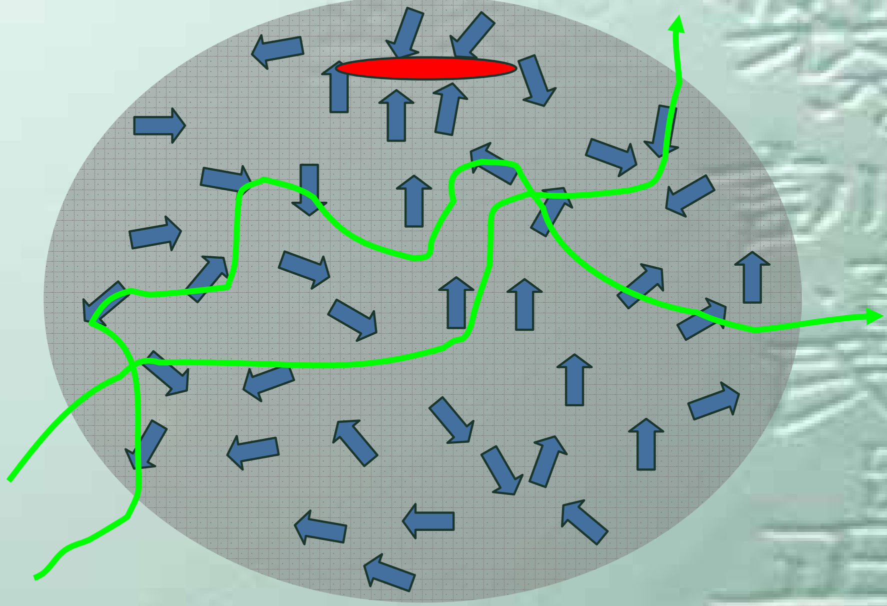
in astrophysical turbulence
— compressibility is important, or flow is supersonic
— magnetic field exists
— observed power spectrum is not that of velocity

Two star formation theories

SF regulated by AD



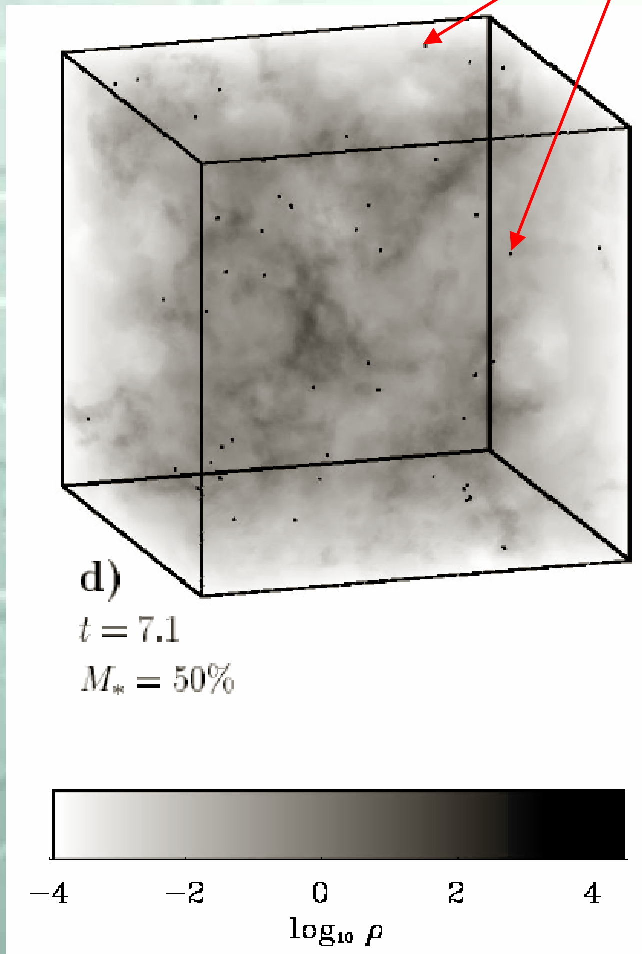
SF regulated by turbulence

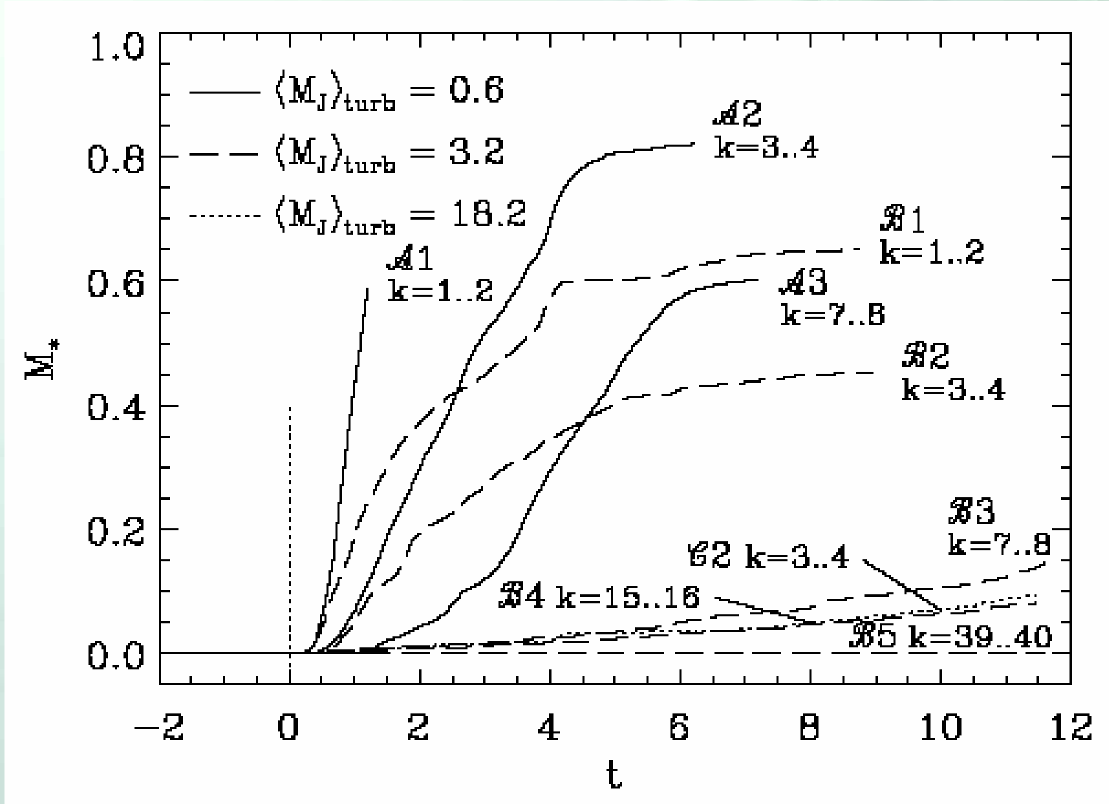


SFEs measured in 3D driven HD turbulent flows

sink particles $\rho > 10^4 \rho_0$

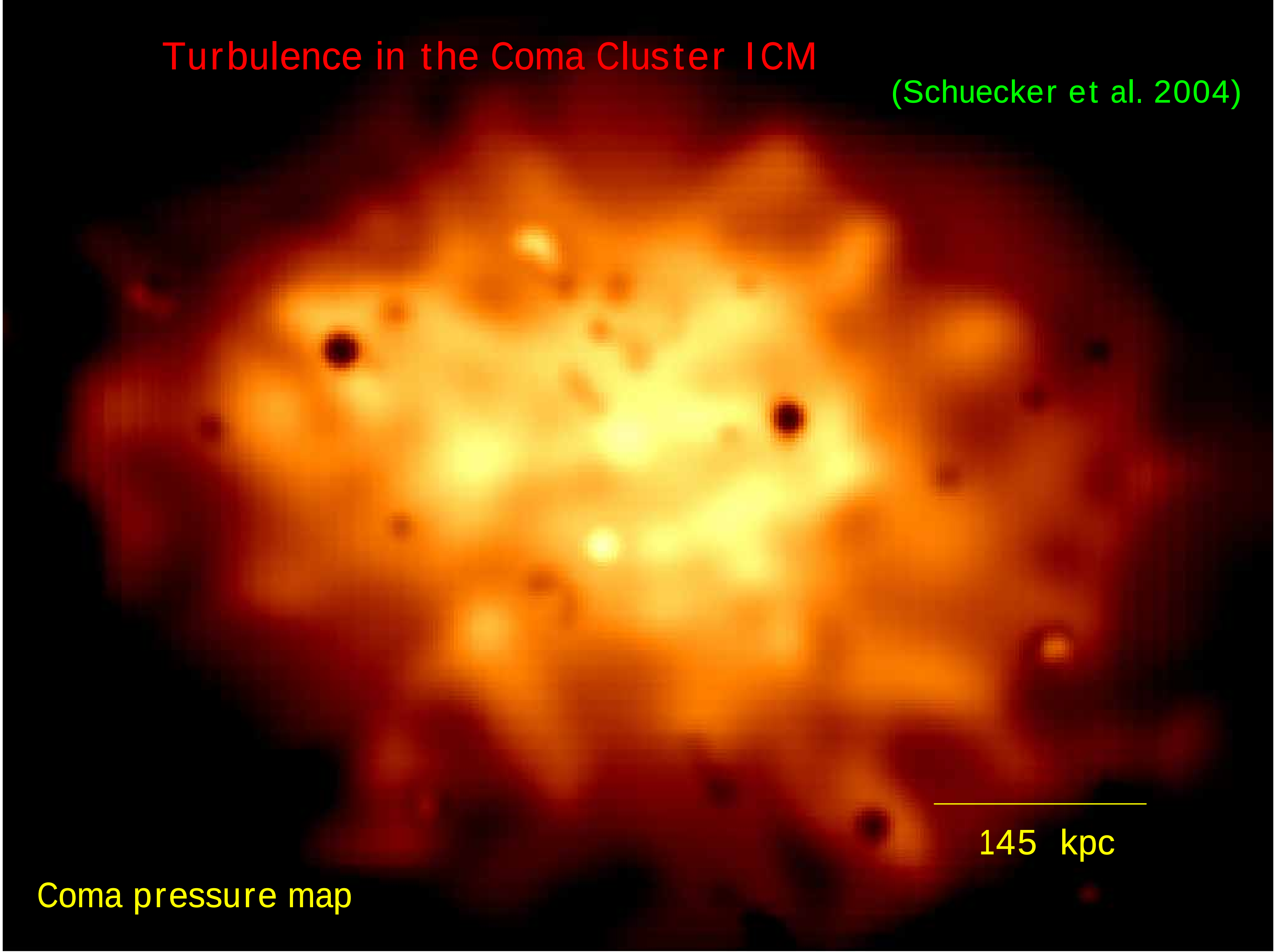
SPH calculations





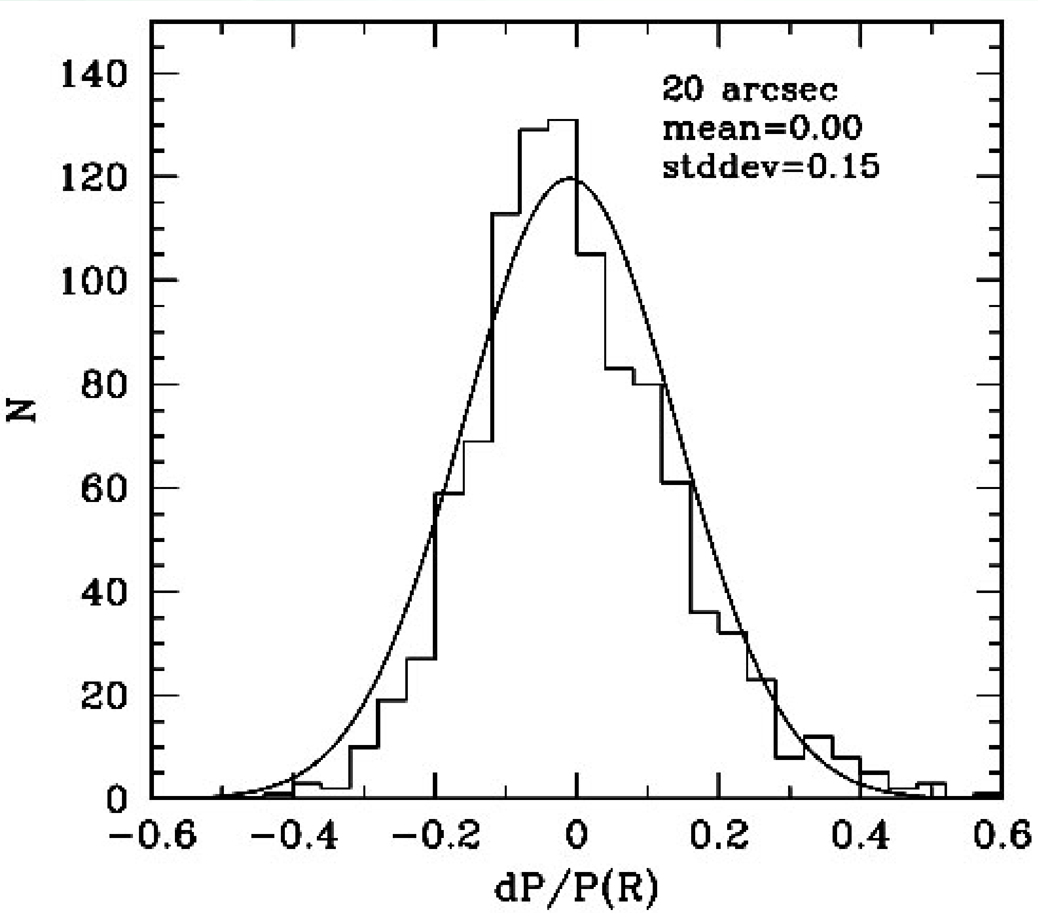
- M_* is mass fraction in sink particles.
- $\langle M_J \rangle_{turb}$ is effective turbulent Jeans Mass
- SFEs are very high in 3D driven HD turbulent flows, except cases driven at small scales.

(Klessen et al. 2000)

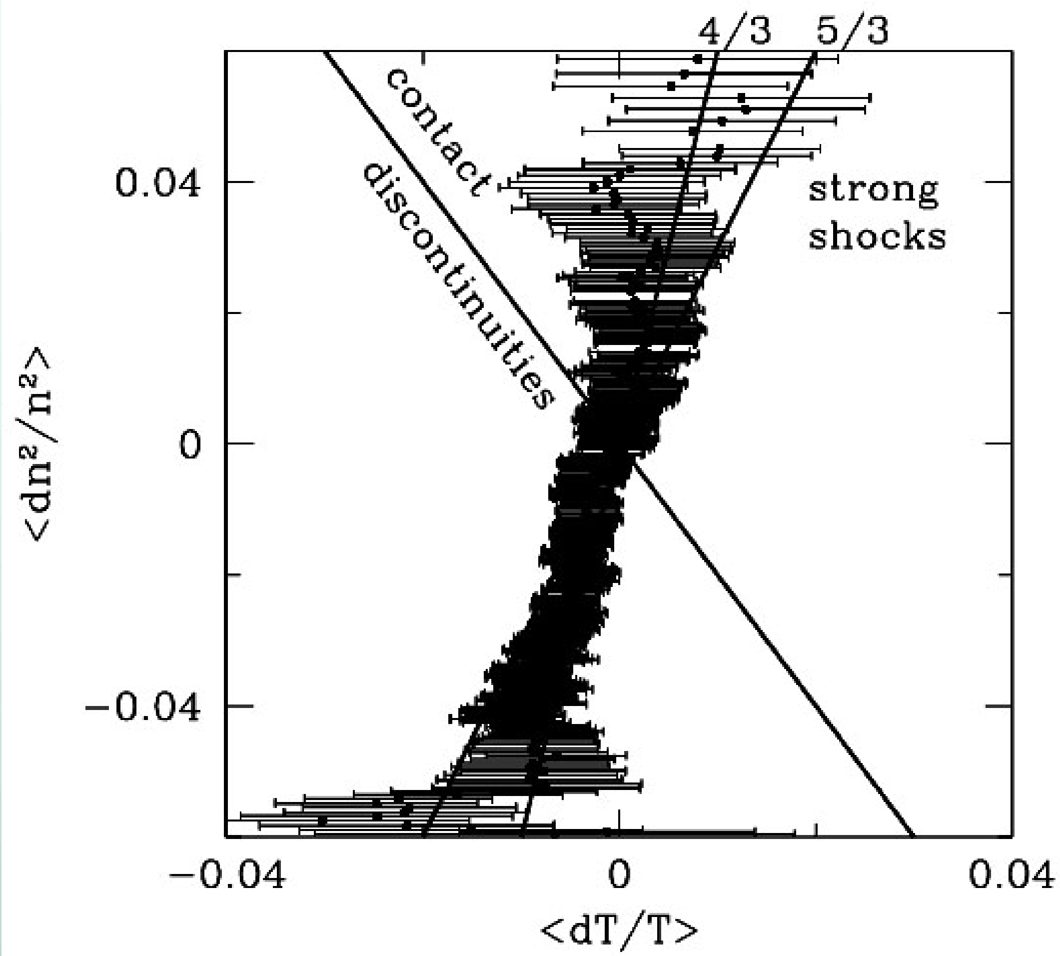

Turbulence in the Coma Cluster ICM
(Schuecker et al. 2004)
145 kpc
Coma pressure map

pressure fluctuations

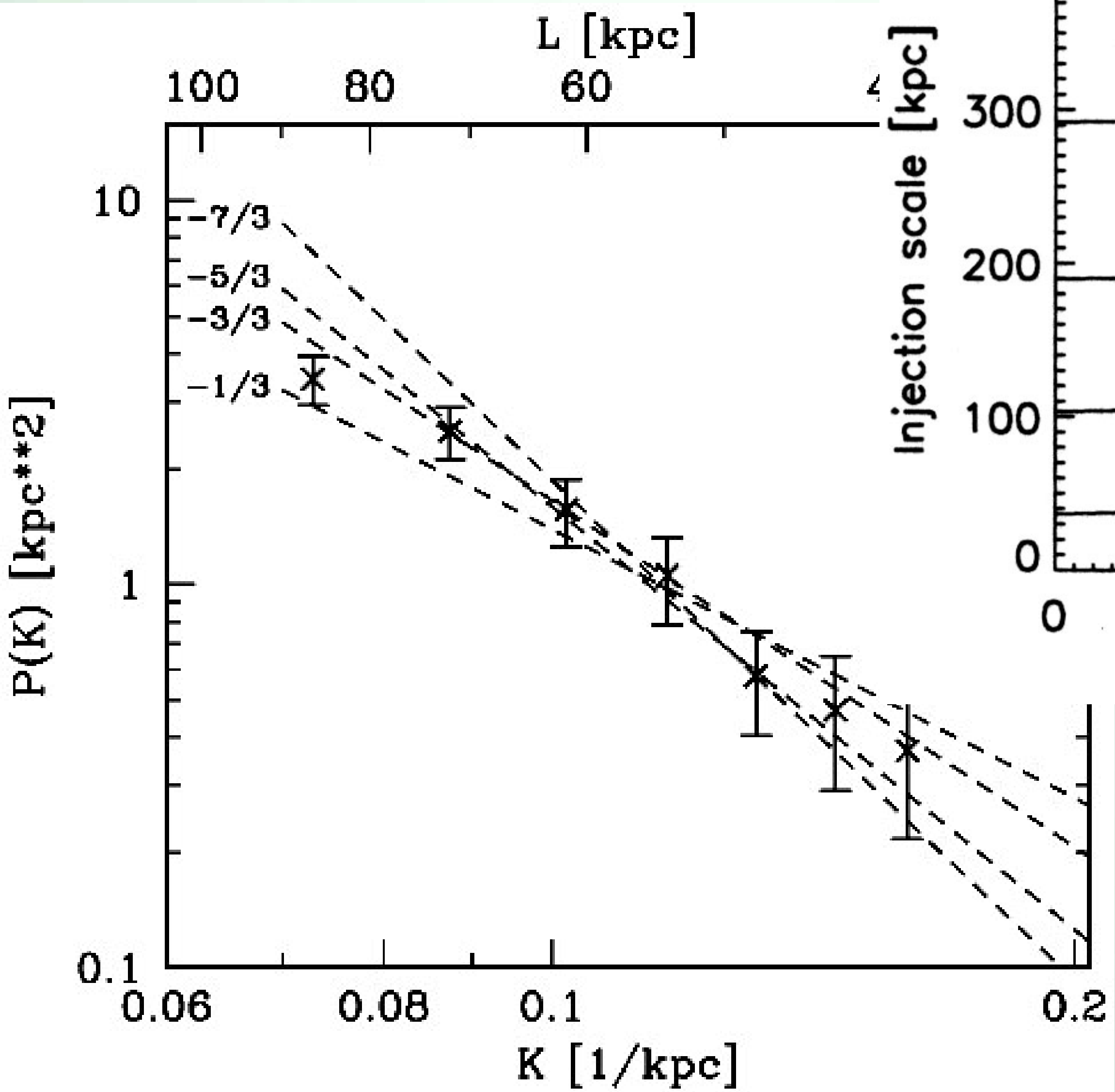
histogram of projected pressure fluctuations

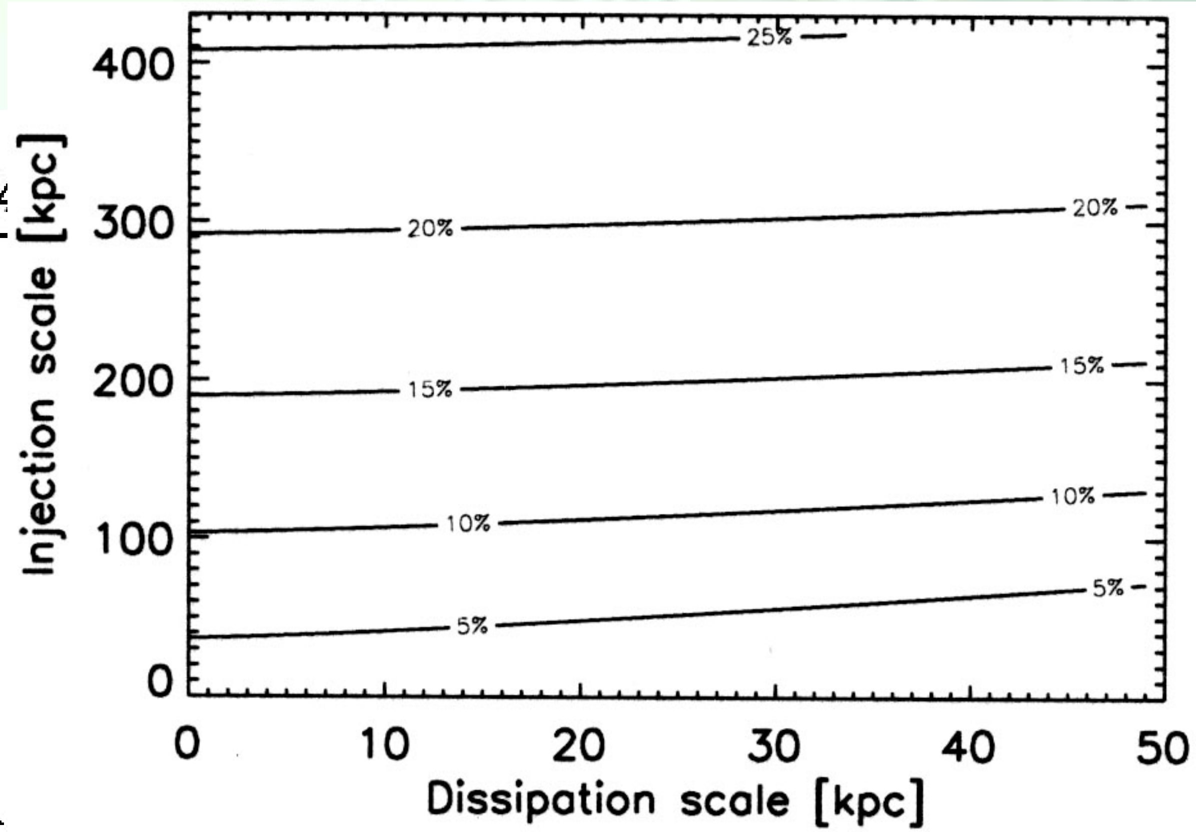


$\delta\rho$ vs δT



fluctuations are mostly gaussian and adiabatic





energy content of turbulence as fraction of thermal energy

⟹ subsonic turbulent

noise subtracted power spectrum of projected pressure fluctuations with slope n ~ -7/3 ... – 5/3

⟹ close to Kolmogorov

Thank you !