Asian winter school on numerical astrophysics
13 March 2006 (@Chiba Univ)

Introductory Review on Magnetohydrodynamic (MHD) Simulations in Astrophysics

Kazunari Shibata (**柴田一成**)
Kwasan and Hida Observatories,
Kyoto University

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contents

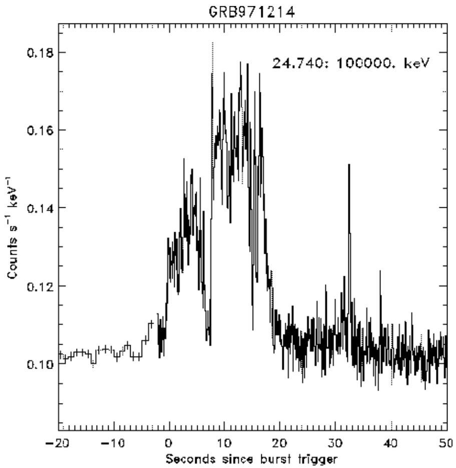
1. Introduction

Why do we need MHD simulations in astrophysics ?

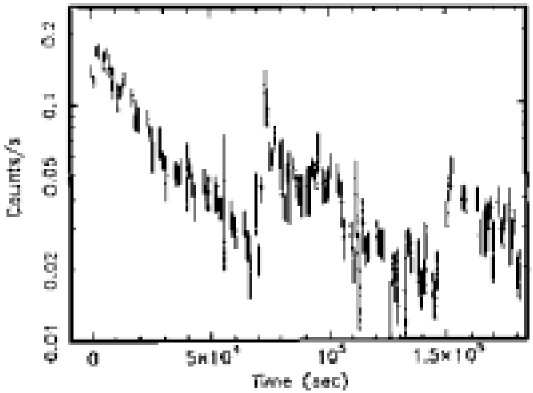
This is because recent astrophysical observations revealed various common active phenomena, such as jets, outflows, flares, bursts, and etc., on widely different scales, ranging from the Sun, stars, galaxies, and even to cluster of galaxies.

Hence the approach based on unified view of these phenomena is necessary, which is hydrodynamic and MHD simulations

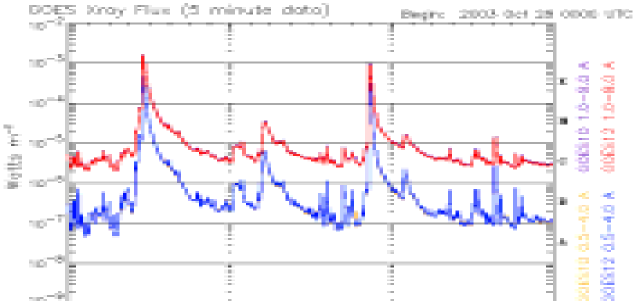
universe is full of flares



Gamma ray bursts

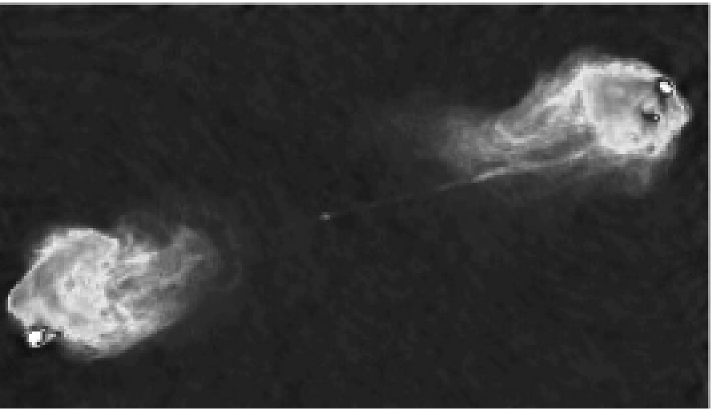


Protostellar flares



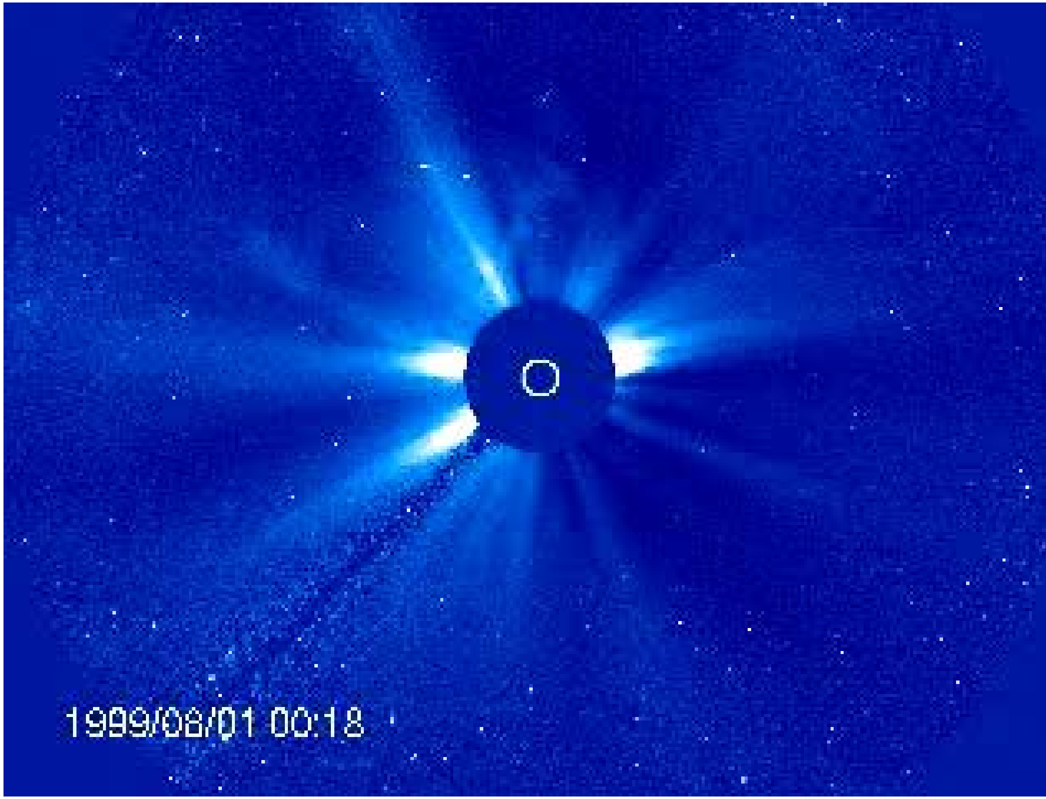
Solar flares

universe is full of jets and mass ejections



AGN (active galactic nuclei) jets

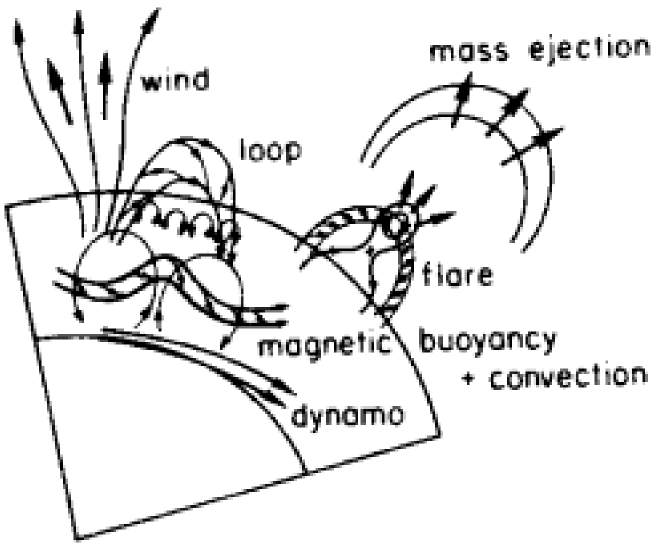
protostellar jets



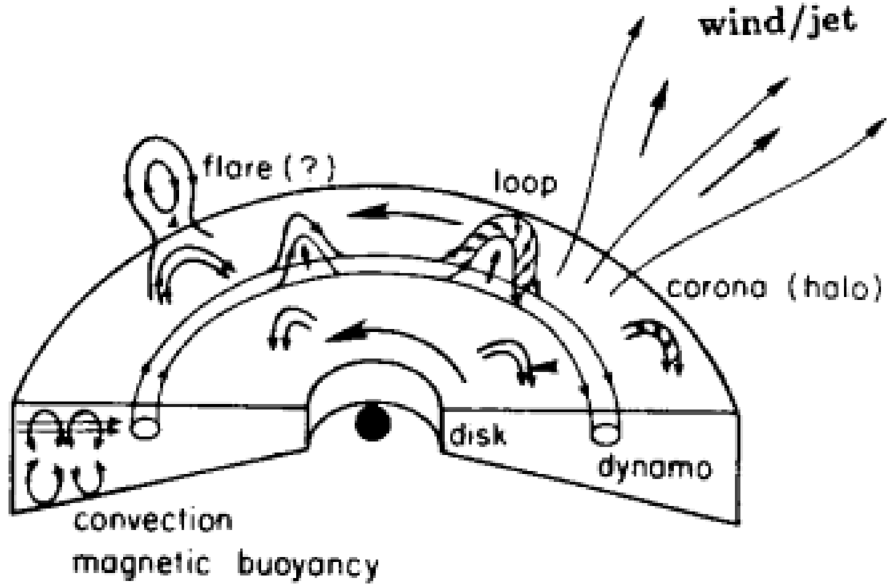
Coronal mass ejections

Solar jet

Basic MHD processes in stars and disks



(stars)



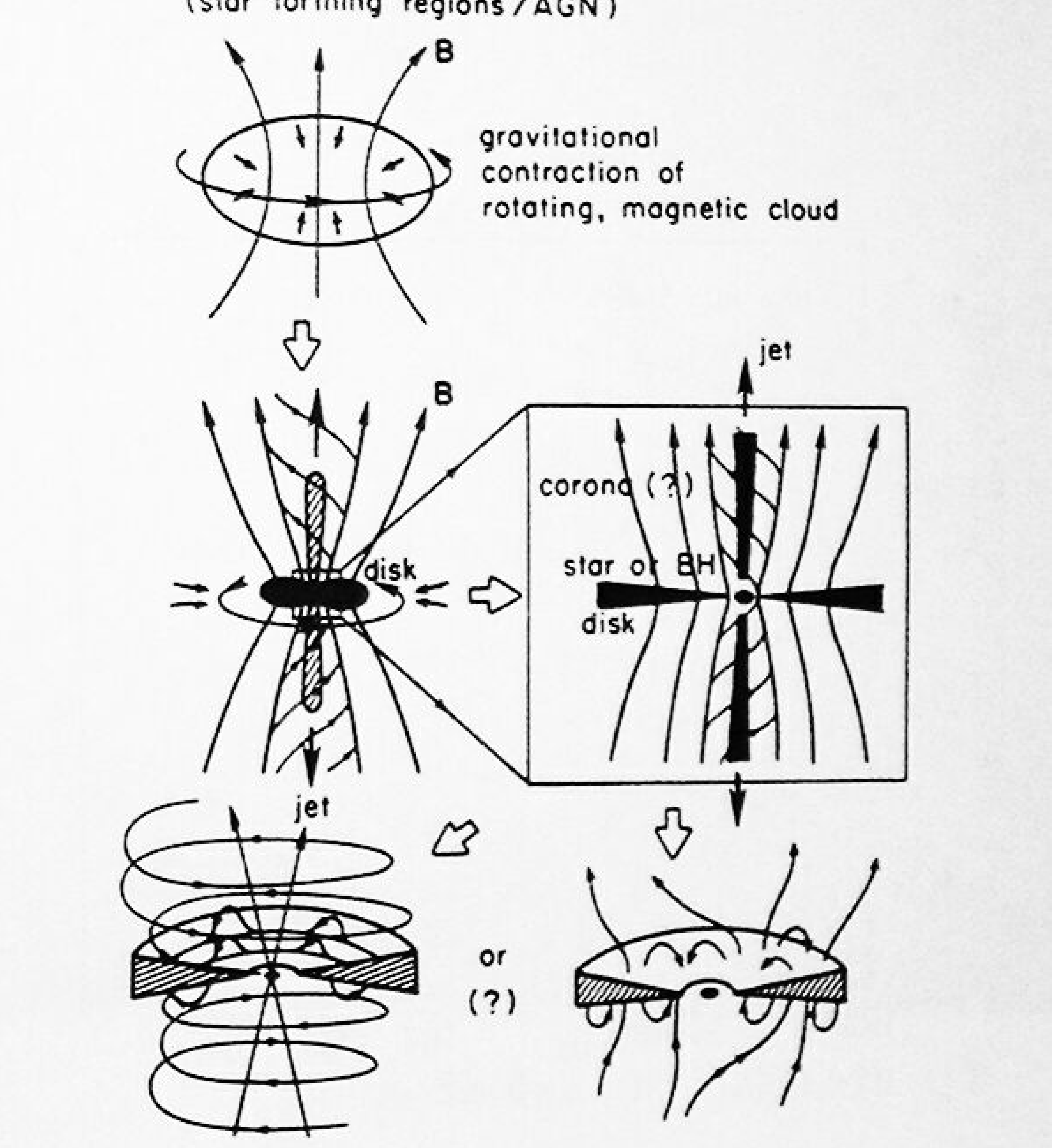
(disks)

From Tajima and Shibata (1997) Plasma Astrophysics

Formation of celestial objects

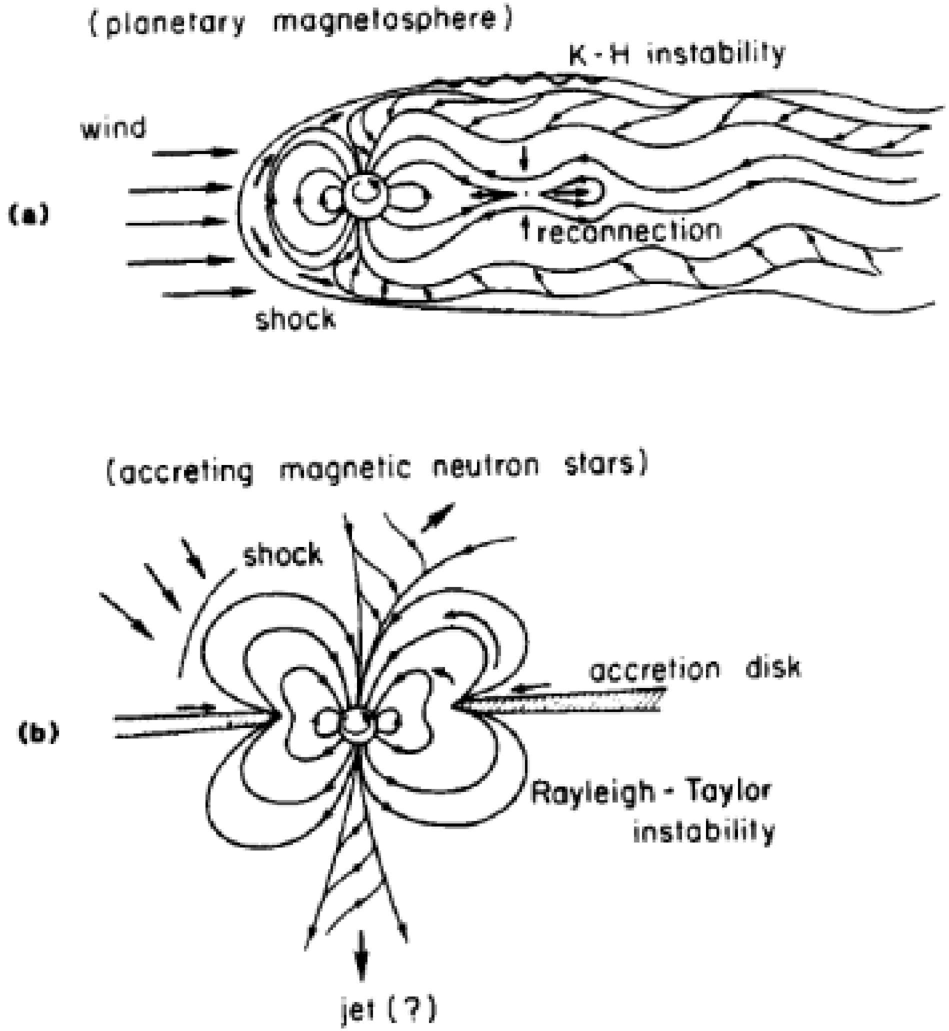
galaxies
stars
planets
supernovae

formation of disk and jet



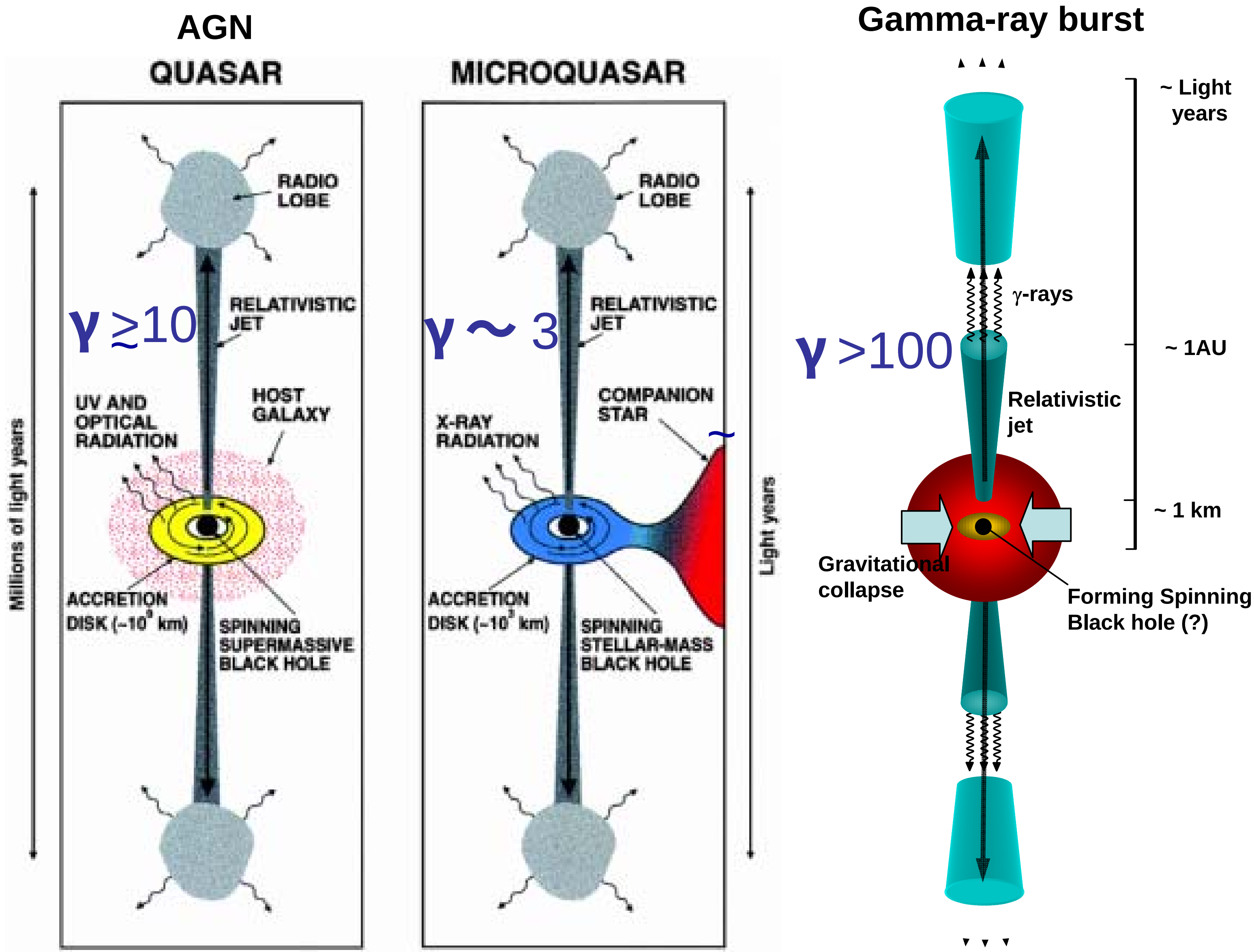
From Tajima and Shibata (1997) Plasma Astrophysics

Magnetospheres of planets and neutron stars



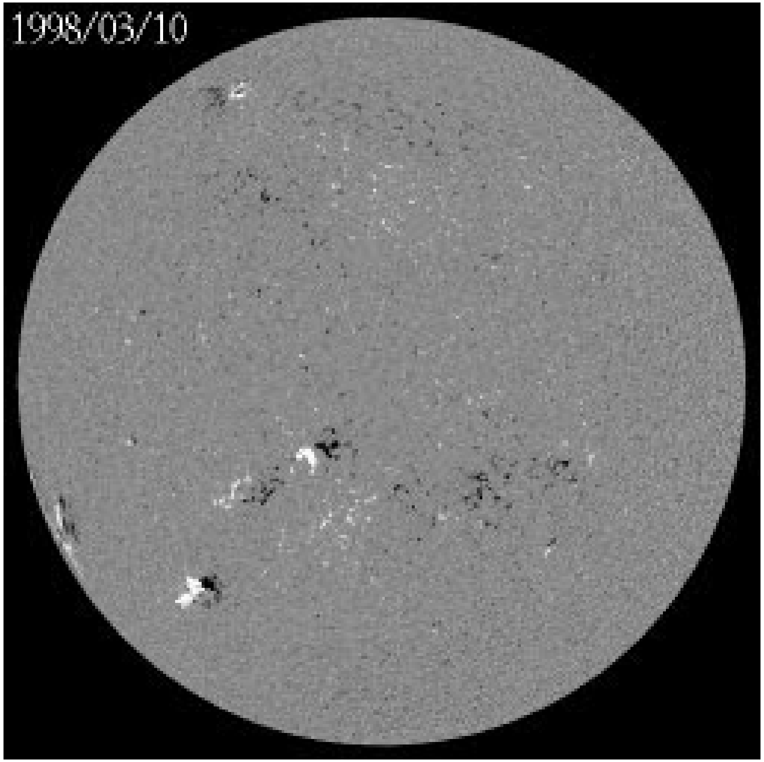
From Tajima and Shibata (1997) Plasma Astrophysics

Relativistic Jets in the Universe

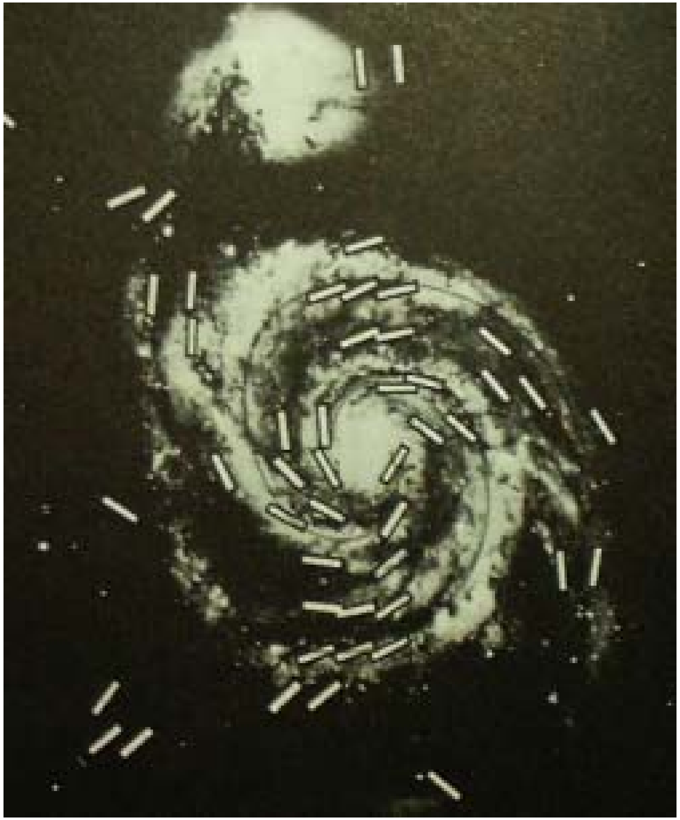


irabel, Rodriguez 1998

Observations of magnetic field



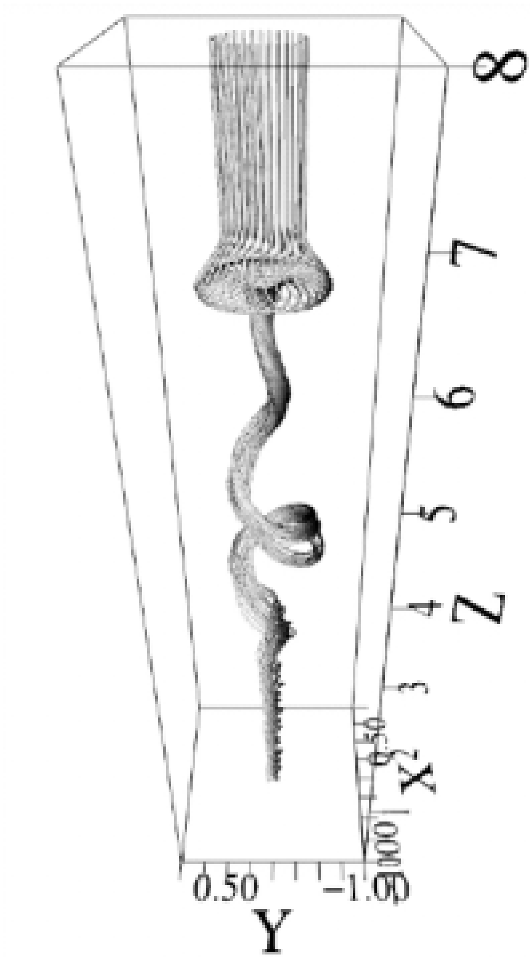
Solar magnetic field (SOHO/MDI)
white-black = positive-negative polarities
B = a few G ~ 3000 G

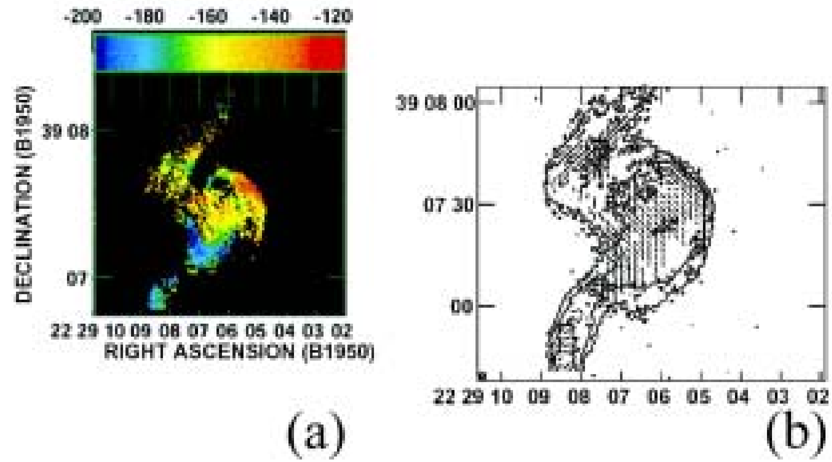


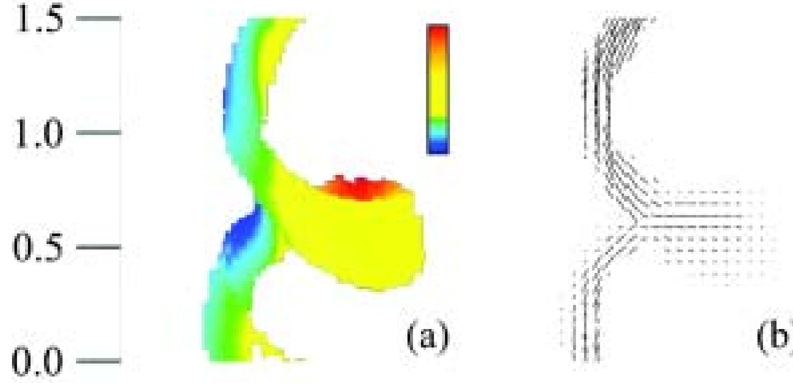
Galactic magnetic field (M51)
(Tosa and Fujimoto 1974)
B = a few micro G

Observation of Faraday rotation measure of AGN(active galactic nuclei) jet suggesting the existence of helical magnetic field in the jet

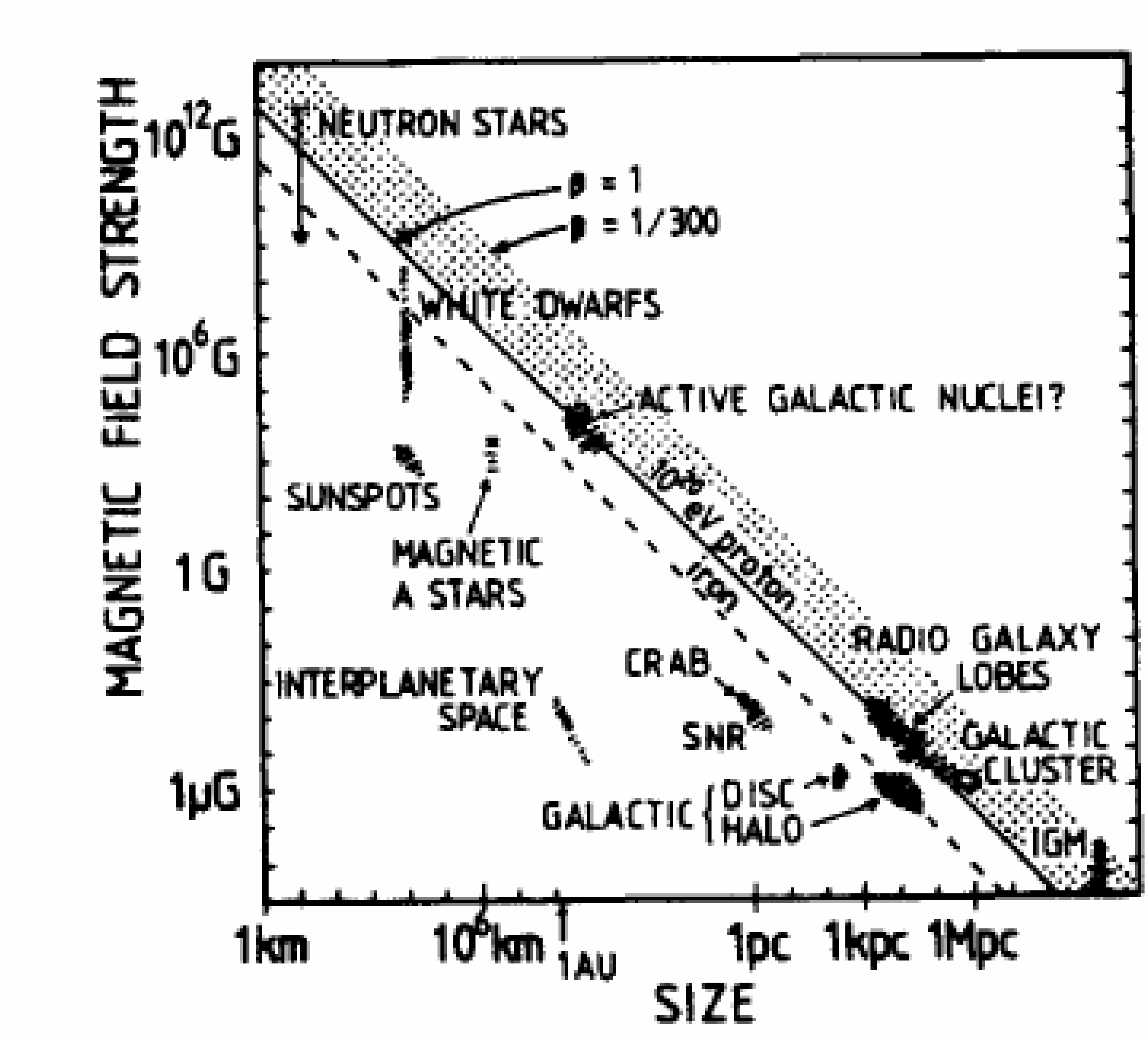
(Kigure et al. 2004)



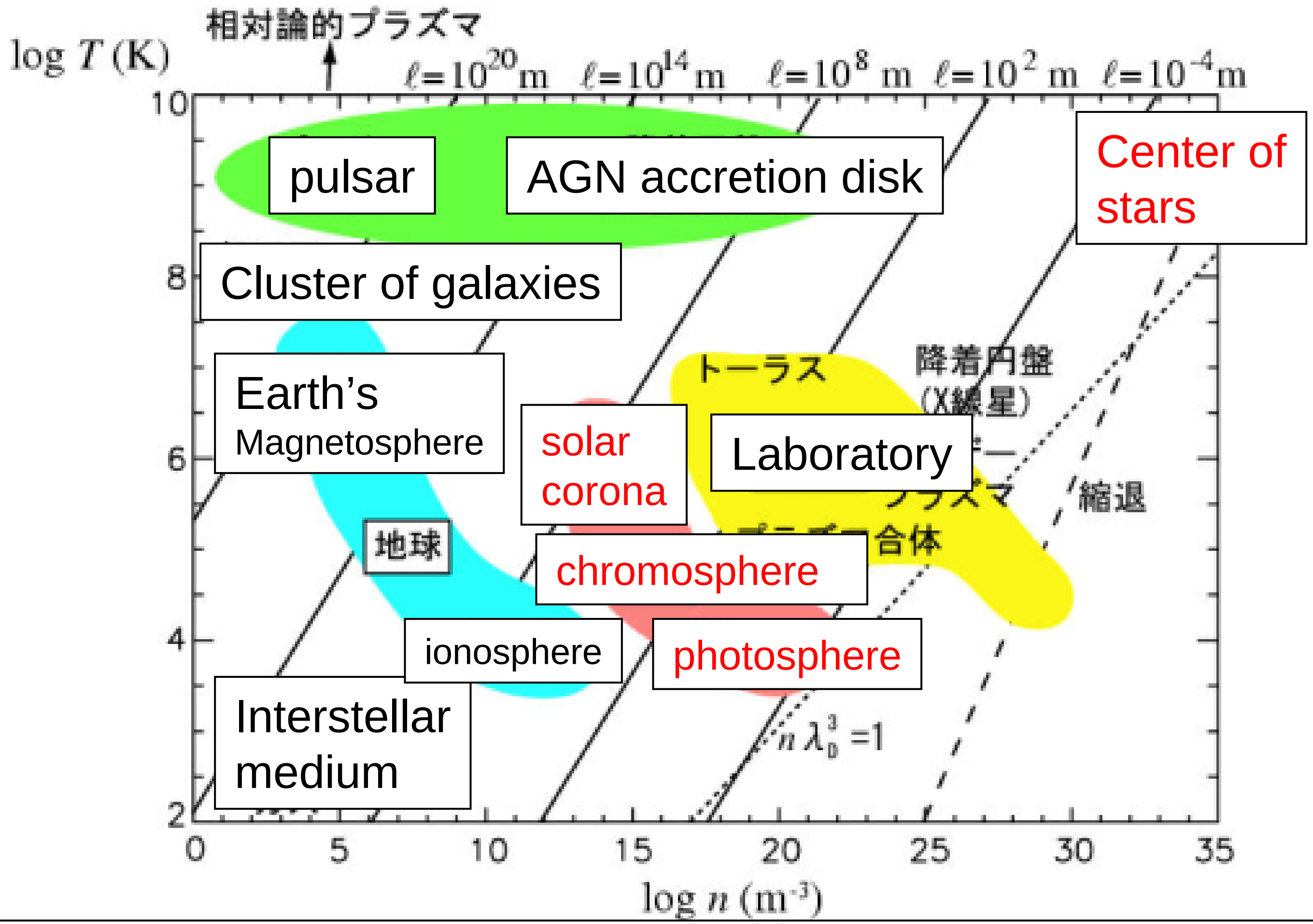




Magnetic fields are ubiquitous in our universe (Hillas 1984)



Temperature-Density diagram for Solar and Cosmic Plasmas



Most of astrophysical objects can be treated as plasma

Fundamental questions

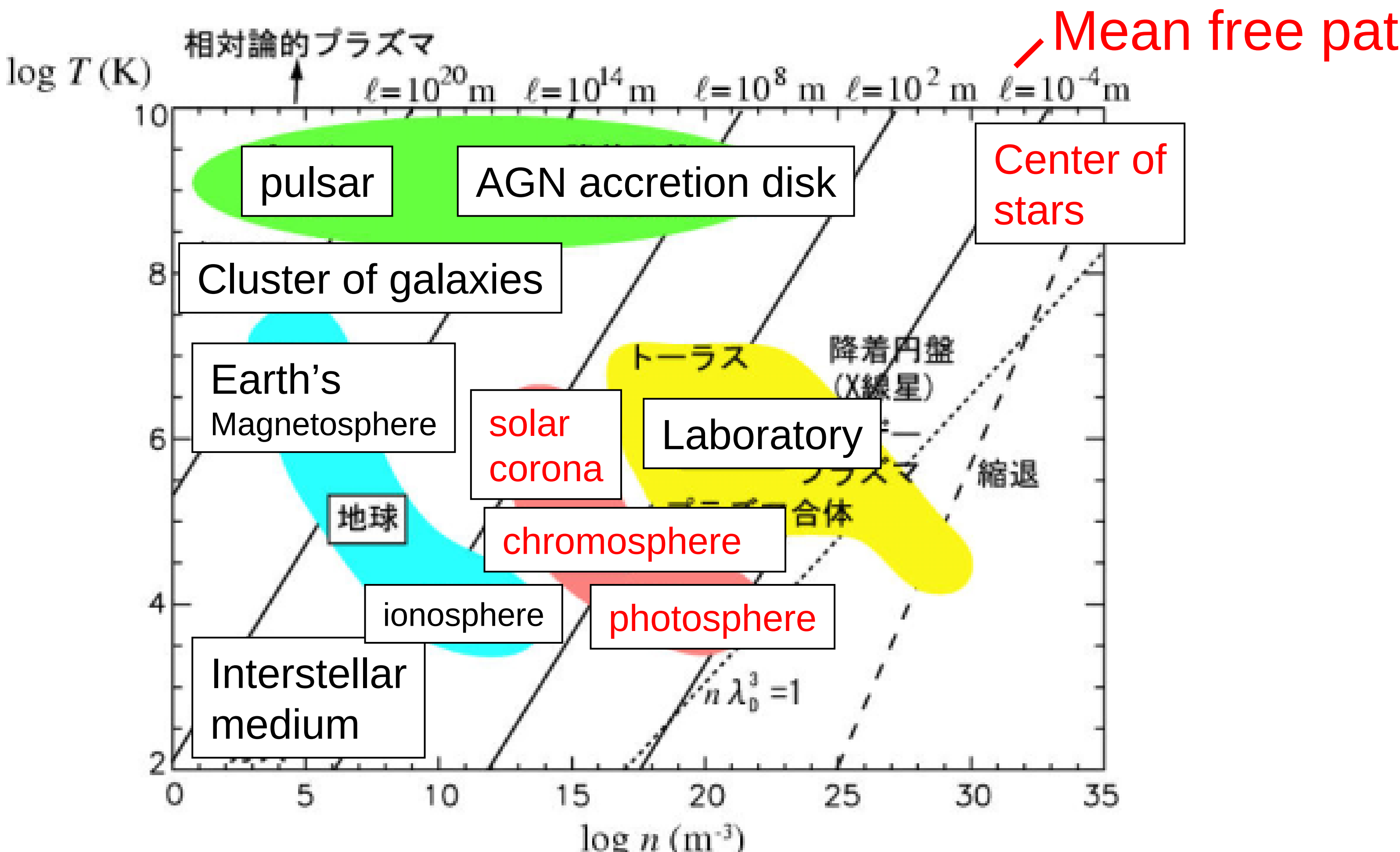
Can we apply hydrodynamics and magnetohydrodynamics (MHD) to these astrophysical phenomena ?

Applicability of Hydrodynamics

- To apply hydrodynamics, we need the condition:
  spatial scale $\gg$ mean free path
  time scale $\gg$ collision time

- These are not necessarily satisfied in many astrophysical plasmas !
  E.g., solar corona, galactic halo, cluster of galaxies,,,

Temperature-Density diagram for Solar and Cosmic Plasmas



Most of astrophysical objects can be treated as plasma

- But in plasmas with magnetic field, the effective mean free path is given by the ion Larmor radius. Hence if the size of the phenomenon is much larger than the ion Larmor radius, hydrodynamic approximation can be used, even if the mean free parth is much longer.

Characteristic length of solar coronal plasma

- Larmor radius

$$r_{Li} = \frac{m_i v c}{eB} \approx 10cm \left(\frac{B}{100G} \right)^{-1} \left(\frac{T}{10^6 K} \right)^{1/2}$$

- Mean free path

$$l_{mfp} = \frac{1}{n} \left(\frac{kT}{e^2} \right)^2 \approx 10^8 cm \left(\frac{T}{10^6 K} \right)^2 \left(\frac{n}{10^9 cm^{-3}} \right)^{-1}$$

- Flare size

$$r_{flare} \approx 10^9 cm$$

MHD approximation

- Hydrodynamic approximation
  characteristic length >> mean free path, or
  ion Larmor radius
- Slow time scale (displacement current is neglected = non-relativistic approx)
  characteristic time >> collision time, or
  ion Larmor period
- Quasi-Neutrality
  particle number density >>
  Goldreich-Julian density >> n_0
  (n = div (v x B)/e)

Applicability of MHD

- **MHD**
  describe macroscopic behavior of plasmas if
  spatial scale $\gg$ ion Larmor radius
  time scale $\gg$ ion Larmor period

- Problems that MHD cannot treat
  - Particle acceleration
  - Origin of resistivity
  - Electromagnetic waves

examples

- <u>Solar corona</u>： $l \sim L$、$r_L << L$

$$l \sim 4\times10^8\left(\frac{T}{10^6 K}\right)^2\left(\frac{n}{10^9 cm^{-3}}\right)^{-1} cm \sim L \sim 10^9 cm$$

$$r_{\mathrm{L,ion}} \sim 10\left(\frac{T}{10^6 K}\right)^{1/2}\left(\frac{B}{100G}\right)^{-1} cm << L \sim 10^9 cm$$

- <u>Cluster of galaxies</u>： What is necessary field strength for $r_L < L$？

$$T \sim 10^8 K,\ \ n \sim 10^{-3} cm^{-3}\ \Rightarrow l \sim 4\times10^{24} cm^{-3}$$

$$L_{cluster} \sim 10Mpc \sim 10^{25} cm,\ L_{galaxy} \sim 10kpc \sim 10^{22} cm$$

$$r_{L,ion} \sim 10^4 B^{-1} cm << L_{galaxy} \sim 10^{22} cm\ \ => B >> 10^{-18} G$$

2．What is astrophysical MHD simulations？

to numerically solve time dependent magnetohydrodynamic (MHD) equations for the purpose of application to astrophysical phenomena

simulation = numerical experiment

- Numerical calculation
  (～numerical simulation)
  ＝third method of science in addition
  to theory and experiment (observation)

- computer＝telescope for theory

Prejudices on astrophysical MHD simulations

1) simulation can solve any problem
2) simulation can yield any disired solution if we assume boundary condition well
3) simulation is easy, and the simulation people are fool
4) simulation deceives people, by showing attractive movies

Be not worried about prejudices

1) simulation is not almighty

2) it is very difficult to control boundary conditions in MHD simulations

3) simulation is not easy, and the simulation people must be clever

4) simulation movies are very useful for research and education

3. Difficulties of Astrophysical MHD simulations

- MHD equations are complicated enough, which are nonlinear partial differential equations with 8 variables
  cf) hydro equations have 5 variables
- Problems become more difficult if resistivity is included because basic physics of magnetic reconnection has not yet been solved

Hydrodynamic equation

(adiabatic、no gravity)

5 unknowns： density（**ρ**）、velocity（**v**）、pressure（**p**）

5 equations： nonlinear partial differential equations

Mass
$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

Momentum
$$\rho \frac{dv}{dt} + \nabla p = 0$$

Energy
$$\rho \frac{d}{dt}\left(\frac{p}{(\gamma - 1)\rho} \right) + p \nabla \cdot v = 0$$

where $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + v \cdot \nabla$

Magnetohydrodynamic（MHD）equation
(adiabatic、no gravity)

8 unknown： density（ρ）、velocity （v）、pressure （p）、 magnetic field （B）

8 equation ： nonlinear partial differential equations

Mass $$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

Momentum $$\rho \frac{dv}{dt} + \nabla p = \frac{1}{c} J \times B$$

Energy $$\rho \frac{d}{dt}\left(\frac{p}{(\gamma - 1)\rho}\right) + p \nabla \cdot v = \frac{1}{\sigma} J^2$$

Induction $$\frac{\partial B}{\partial t} = rot(v \times B - \frac{c}{\sigma} J)$$

where $J = \frac{c}{4\pi} rot\, B$

Common properties of astrophysical plasmas
(difficult to dissipate magnetic field)

Magnetic diffusion time (current dissip. time)

$$t_D = L^2 / \eta \approx 10^{14} L_9{}^2 T_6{}^{3/2} \;\; s$$

$$\eta = \eta_{Spitzer} \approx 10^4 T_6{}^{-3/2} \;\; cm^2 /$$

Flare time

$$t_{flare} = 10^2 - 10^3 \sec$$

Alfven time

$$t_A = L / V_A = 10 \sec$$

Magnetic Reynolds number

$$R_m = t_D / t_A \approx 10^{13} >> 1$$

Hence, ideal MHD approx. is assumed

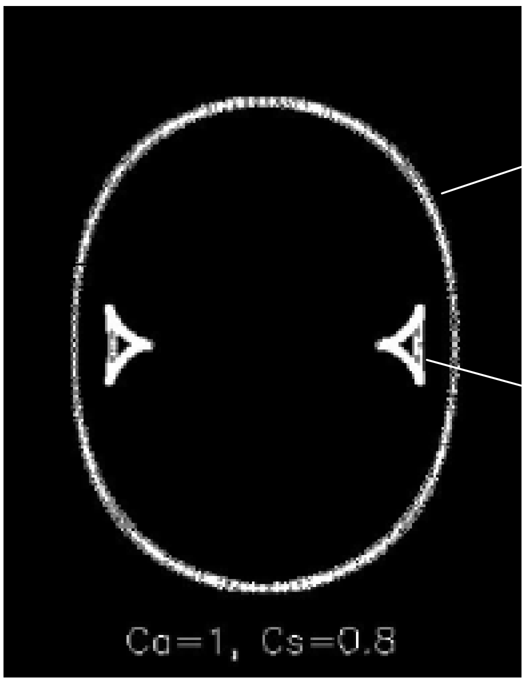
Difference between hydro and MHD

hydro	MHD
5 variables	8 variables
acoustic wave	fast mode
	slow mode
	Alfven mode

MHD waves (Alfven, fast, slow)

$$\omega^2 - k^2 V_A{}^2 \cos^2\theta = 0$$

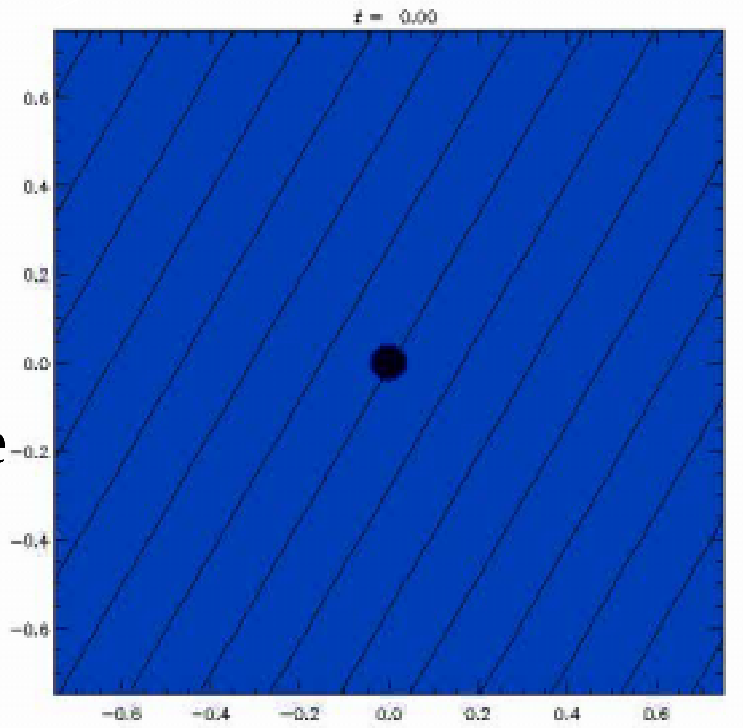
$$\omega^4 - (C_s{}^2 + V_A{}^2)k^2\omega^2 + 4C_s{}^2 V_A{}^2 k^4 \cos^2\theta = 0$$



lagnetic
eld

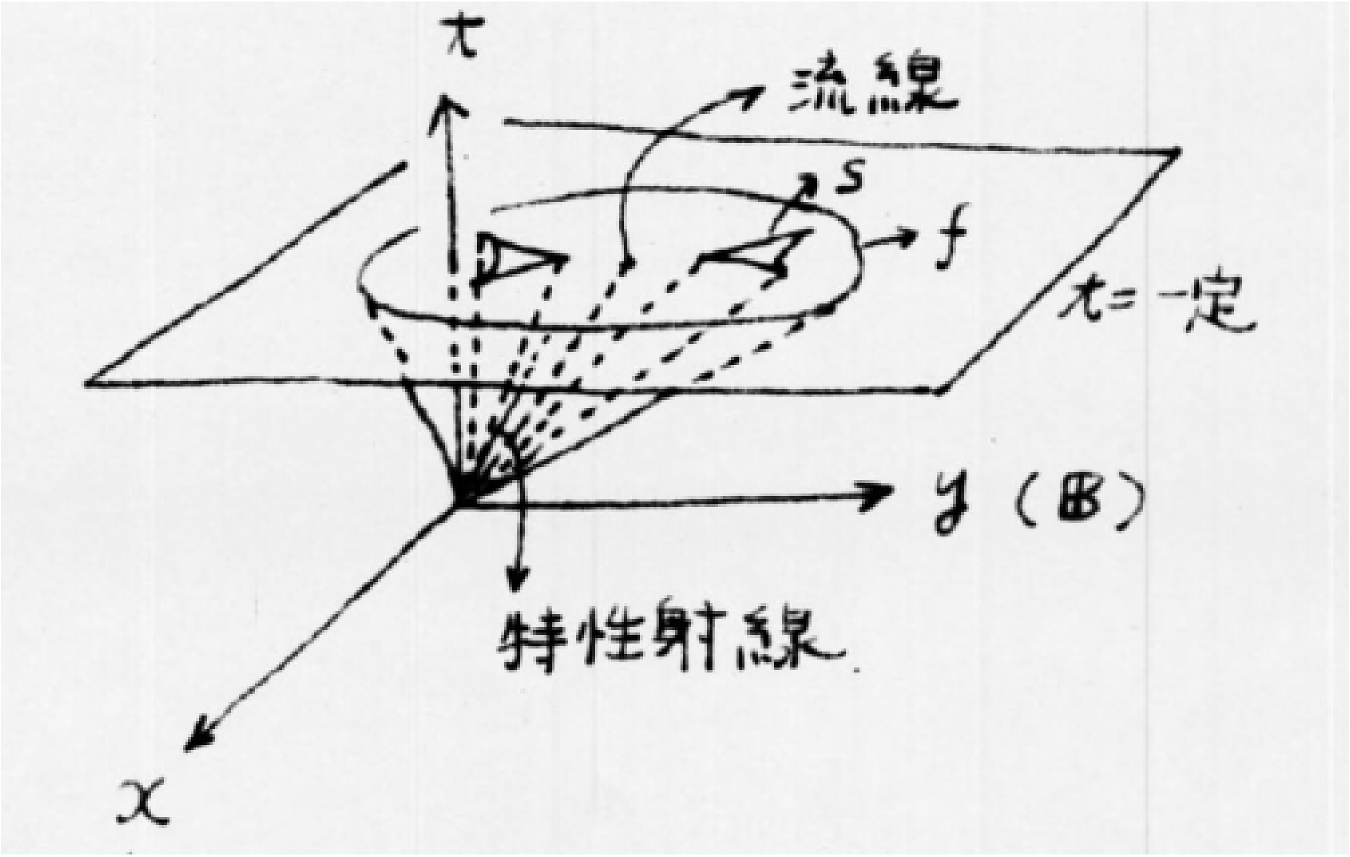
Fast mode

Slow mode



Group velocity diagram $v_g = \partial\omega / \partial k$

MHD wave characteristics



2D： fast ＋－, slow ＋－, stream line＝ 5 bicharacteristics
2.5D, 3D：
fast ＋－, Alfven ＋－, slow ＋－, stream line＝7 bicharacteristics

method of numerical MHD

- MHD equations become normal compressible hydro equation if magnetic field = 0
- Ideal hydro equations = hyperbolic partial differential equations
- Similarly, ideal MHD equations are also hyperbolic partial differential equations
- Hence various numerical methods developed for hydrodynamics are applicable to MHD equations

Difference method

- Difference method： differential is approximated by difference. Finite number of grid points are used. (detailed explanation **=>** following lectures)

$$\frac{\partial \rho}{\partial t} \Rightarrow \frac{\rho_{n+1} - \rho_n}{\Delta t}$$

- Particle method： eq. of motion of super particle are solved. It is the Lagrangian method. It is not suitable for MHD, since grid points are needed for solving induction equation

Advanced Methods

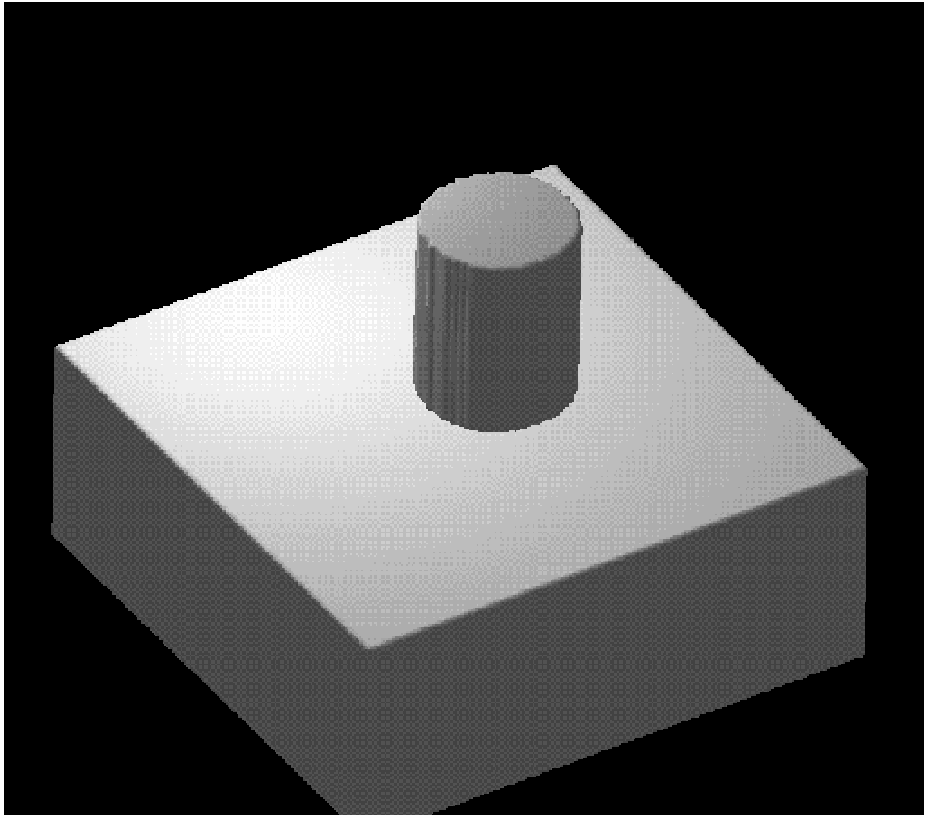
- Approximate Rieman solver (good at shocks)
- CIP (-MOCCT) method (good at handling contact discontinuity and multi-phase matter)
- Spectral method (good at problems with periodic boundaries)

CIP scheme

(Constrained Interpolation Profile/
Cubic Interpolated Pseudoparticle)

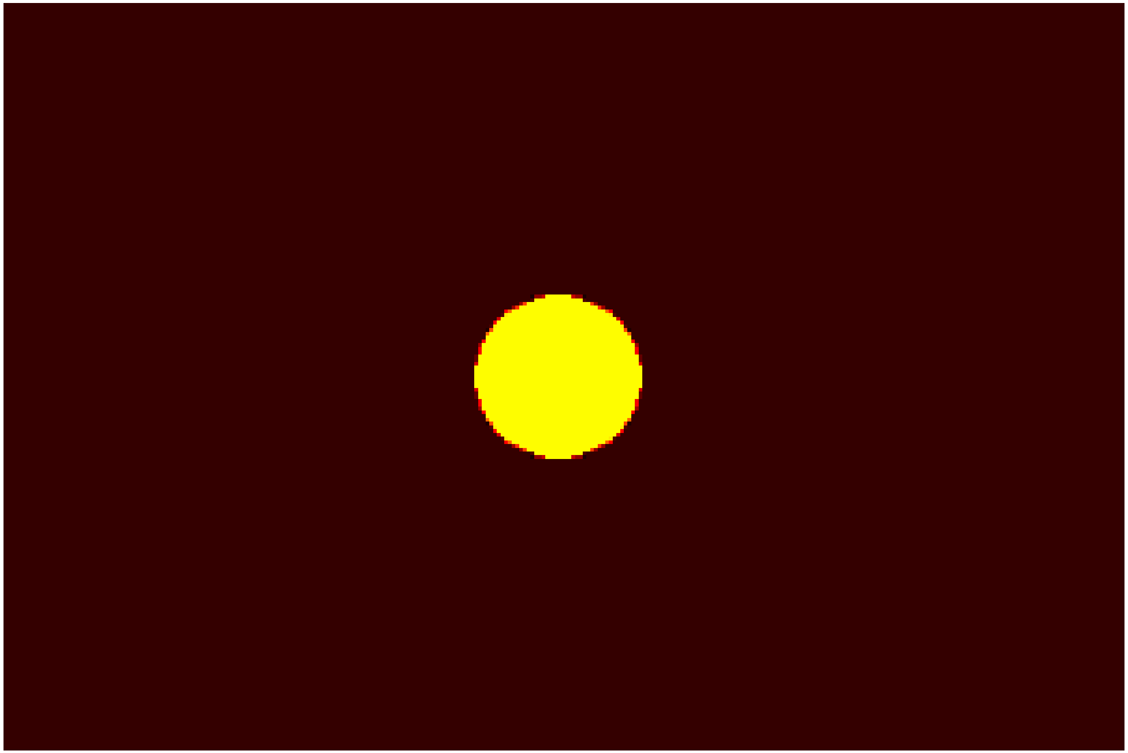
Prof. T. Yabe invented in 1991

good at contact discontinuity,
can solve gas, liquid, and solid simultaneously



Example with CIP scheme

Comet Shoemaker-Levy 9 on entry into Jovian atmosphere (Yabe et al. 1994)



Why astrophysical hydro/MHD simulations are difficult ?

- There is a gravity
  - Hence, dynamic range becomes huge

    => large density variation
- There is no boundary
  - Both leads to supersonic flow

    => strong shocks
- Size scale is huge
  - leading to large Reynolds number

    => strong turbulence

Historical examples :

one of the first hydro simulations of supernova

(Colgate and White1966 ApJ 143, 626-681)

THE HYDRODYNAMIC BEHAVIOR OF SUPERNOVAE EXPLOSIONS*

STIRLING A. COLGATE AND RICHARD H. WHITE
Lawrence Radiation Laboratory, University of California, Livermore, California

Received June 29, 1965

ABSTRACT

We regard the release of gravitational energy attending a dynamic change in configuration to be the primary energy source in supernovae explosions. Although we were initially inspired by and agree in detail with the mechanism for initiating gravitational instability proposed by Burbidge, Burbidge, Fowler, and Hoyle, we find that the dynamical implosion is so violent that an energy many times greater than the available thermonuclear energy is released from the star's core and transferred to the star's mantle in a supernova explosion. The energy released corresponds to the change in gravitational potential of the unstable imploding core; the transfer of energy takes place by the emission and deposition of neutrinos.

I. INTRODUCTION

The original concept of Burbidge, Burbidge, Fowler, and Hoyle (1957; hereinafter referred to as "B²FH") for the explosion of a supernova depended upon the ingenious observation that the matter of a massive star ($M \gtrsim 10\, M_\odot$) at the end point of its evolution is gravitationally unstable and necessarily initiates a dynamical implosion. It was suggested in B²FH and later discussed in detail by Hoyle and Fowler (1960) that the rapid compression of the implosion triggers a thermonuclear explosion in the envelope which then leads to a major mass ejection from the star. Recently, Ono and co-workers (Ono, Sakashita, and Yawazaki 1960*a*, *b*; Ono, Sakashita, and Ohyama 1961) and Ohyama (1963) have contributed to this concept by calculating analytically the be-

Supernova : 1D gravitational collapse of a stellar core and subsequent explosion (Colgate and White 1966 ApJ)

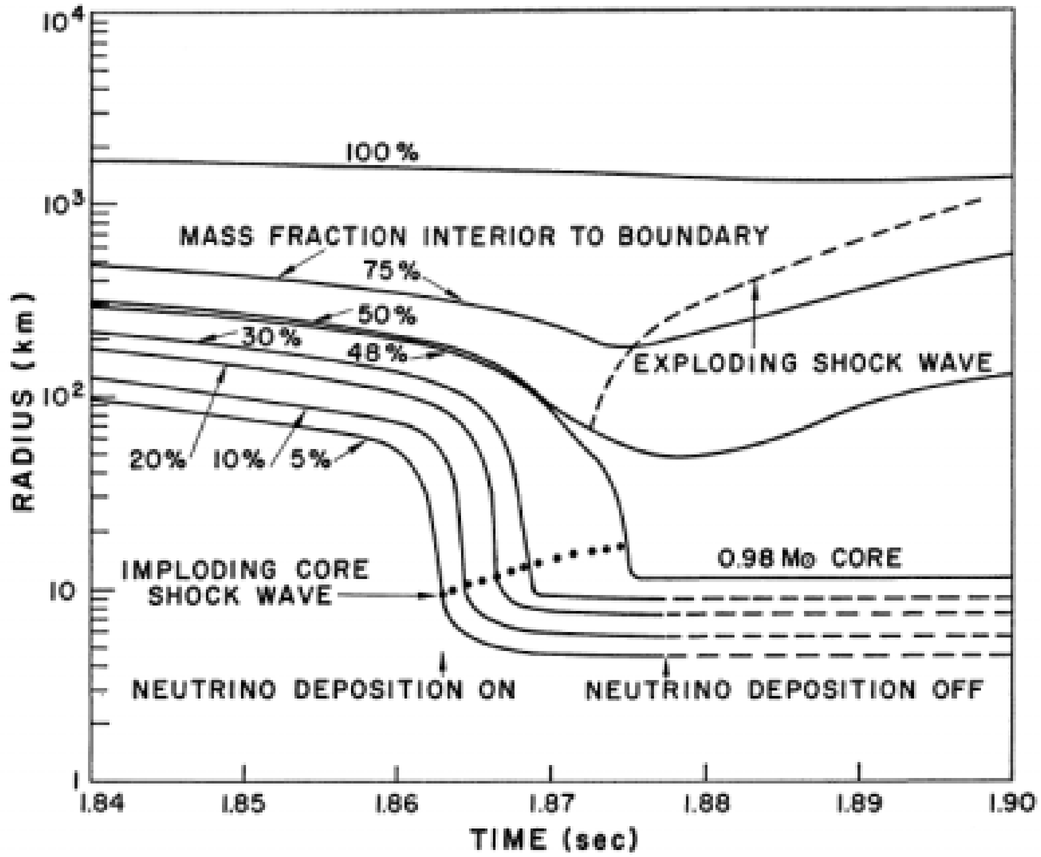


FIG. 30.—2 $M_\odot$ supernova radius versus time with neutrino deposition. The instability occurs due to neutrino emission and nucleon binding in the equation of state with $\rho > 2 \times 10^{11}$ gm/cm^3.

334 citations

(Colgate and White 1966)

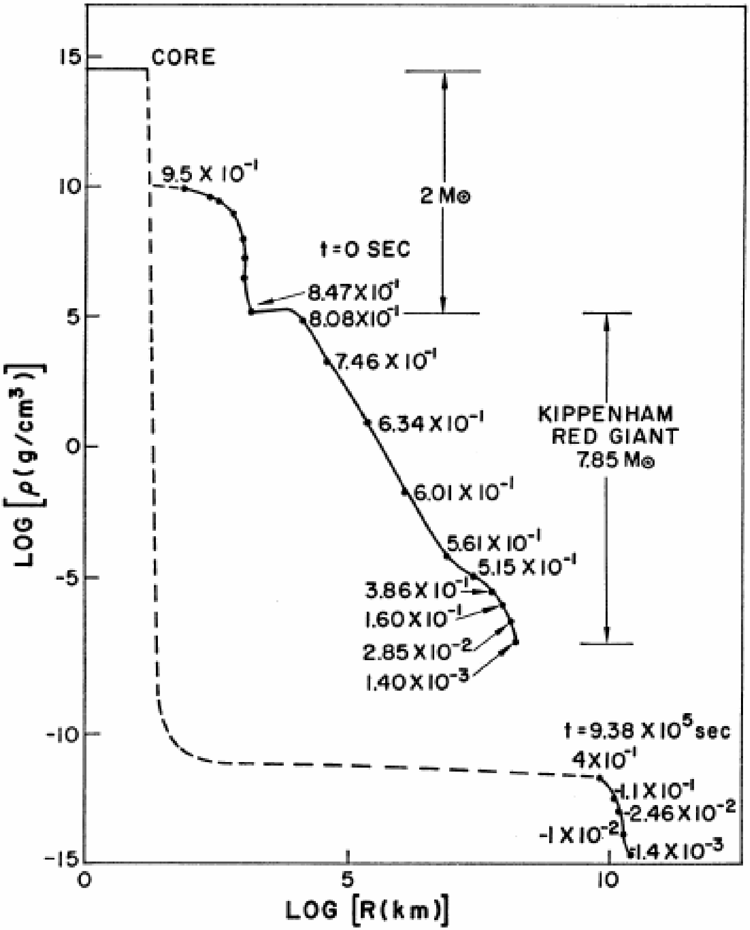


FIG. 37.—Red-giant structure log density versus log radius. The envelope has been "tacked" on to the 2 $M_\odot$ supernova at the time of explosion, giving 9.5 $M_\odot$ total.

Density decreases
By more than 20
Order of magnitude

(Colgate and White 1966)

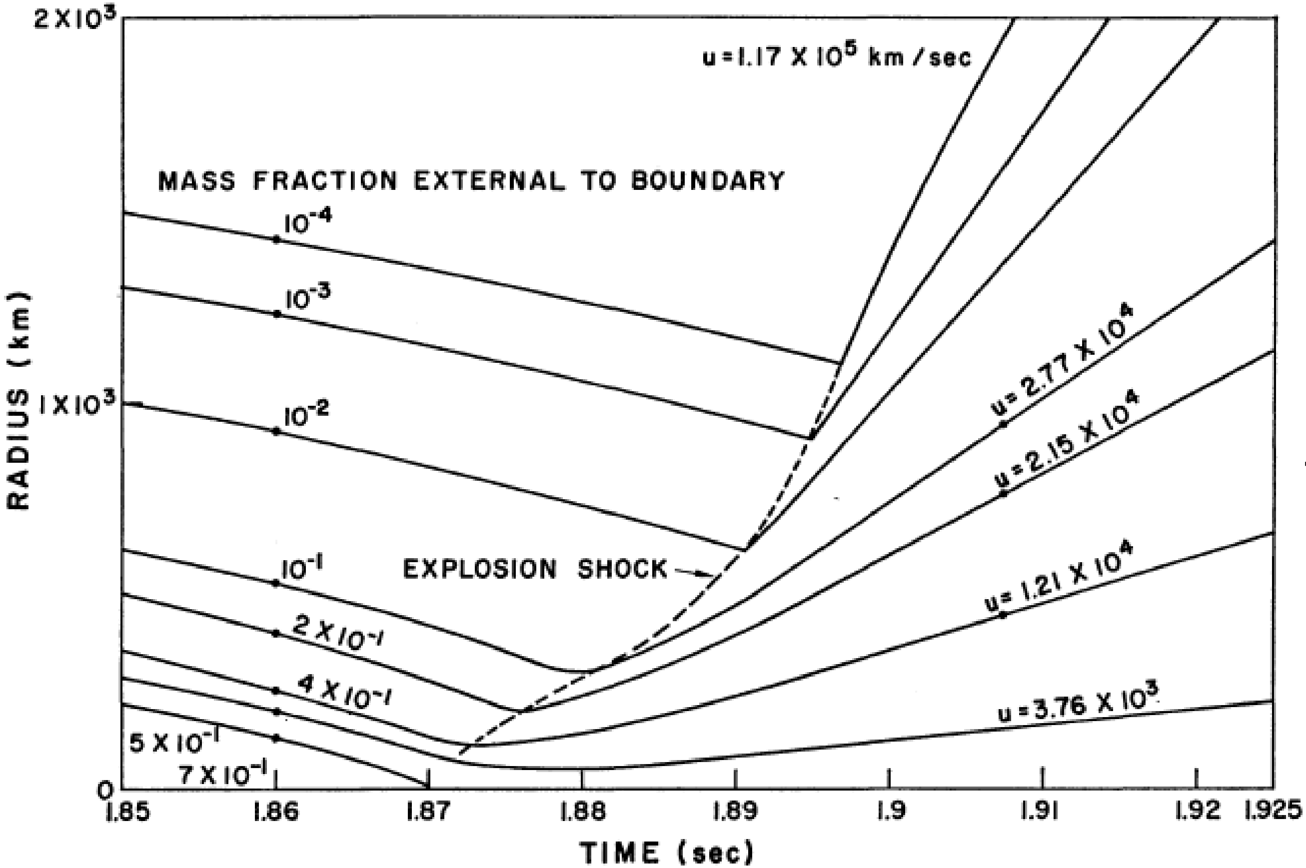


Fig. 31.—2 $M_\odot$ supernova radius versus time with neutrino deposition

Only 1 %
of gravitationa
energy goes
to supernova
explosion

It is interesting to note that supernova explosion
has not yet been solved !

Basic physics of how waves/shocks are amplified

- Amplitude of acoustic wave/shock propagating upward grows in a stratified gas layer because

$$\rho V_{\parallel}^{2} C_s A = \text{constant}$$

$$\therefore\ V_{\parallel} \propto \rho^{-1/2} A^{-1/2} C_s^{-1/2}$$

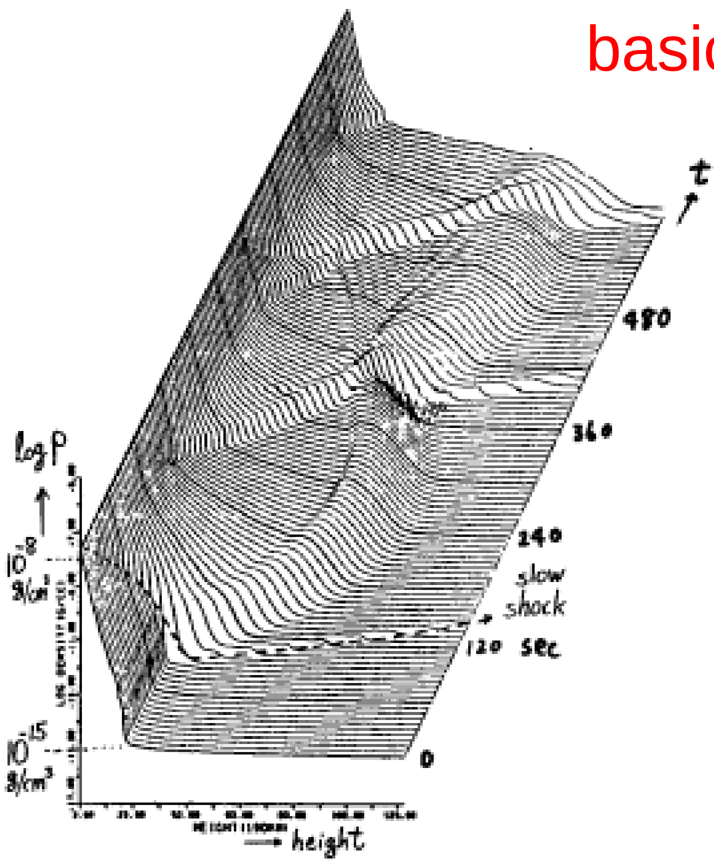
<= Huge density variation

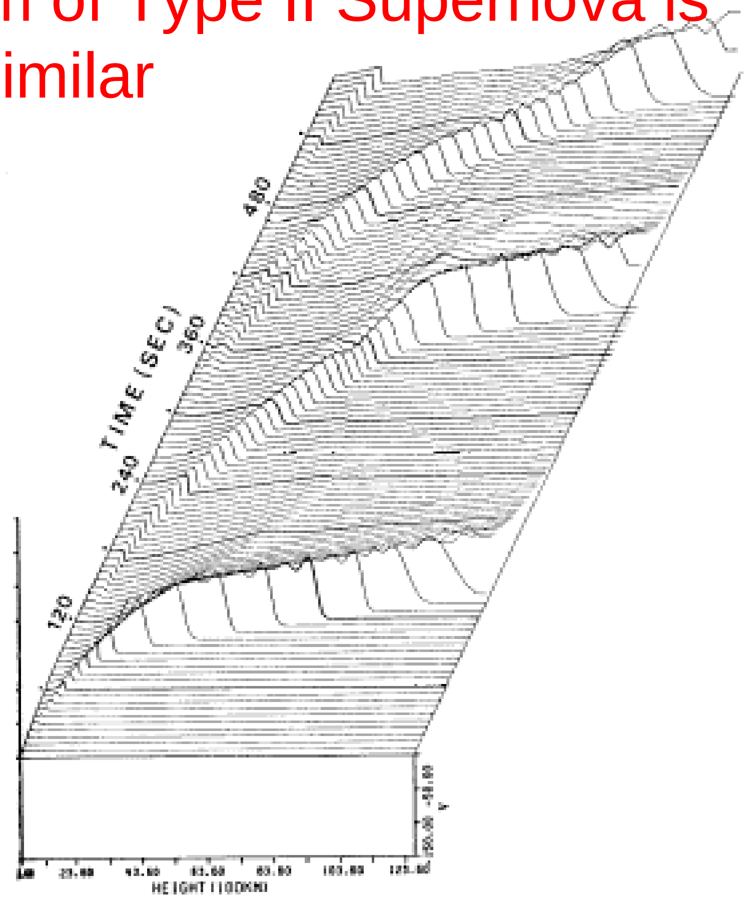
（ ρ ：density、**A**：cross-section of a wave front、**Cs**：sound speed

- Hence, even small amplitude waves become large amplitude waves when they propagate into the upper atmosphere, so shock waves are easily created

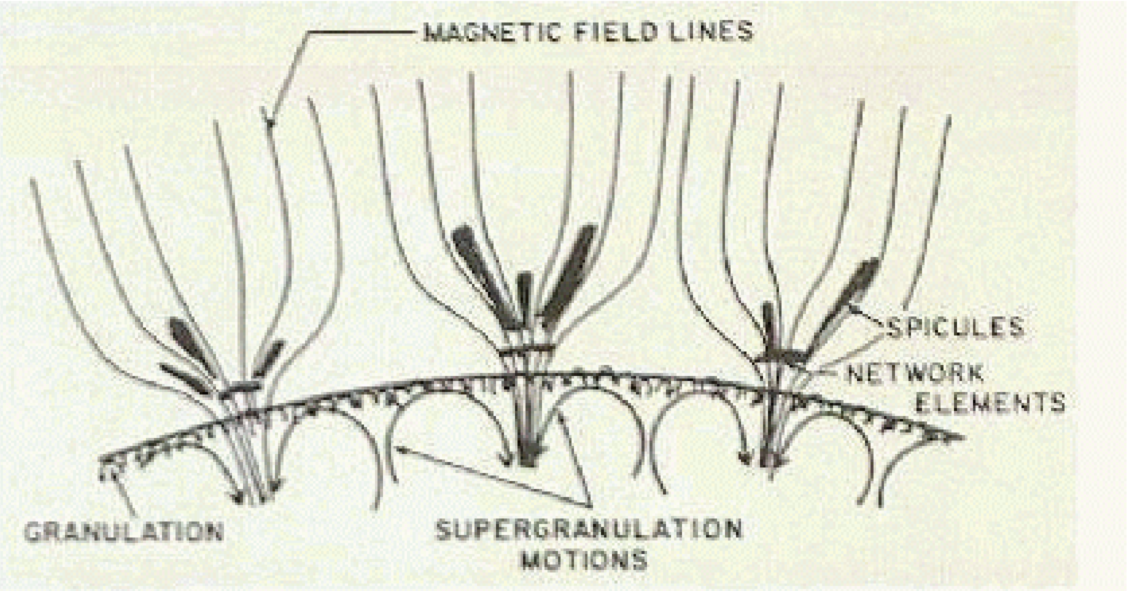
Amplification of Slow mode MHD wave along vertical flux tube (Suematsu et al. 1982. Shibata and Suematsu 1982)

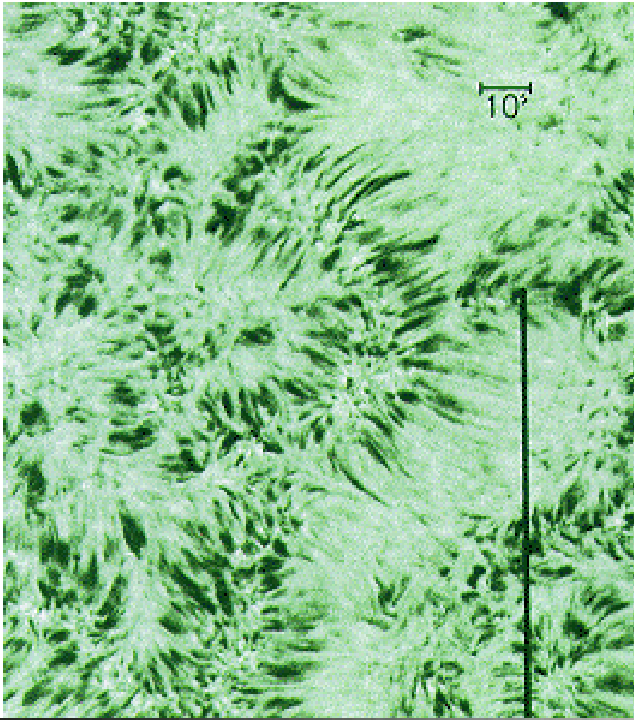
Mechanism of Type II Supernova is basically similar





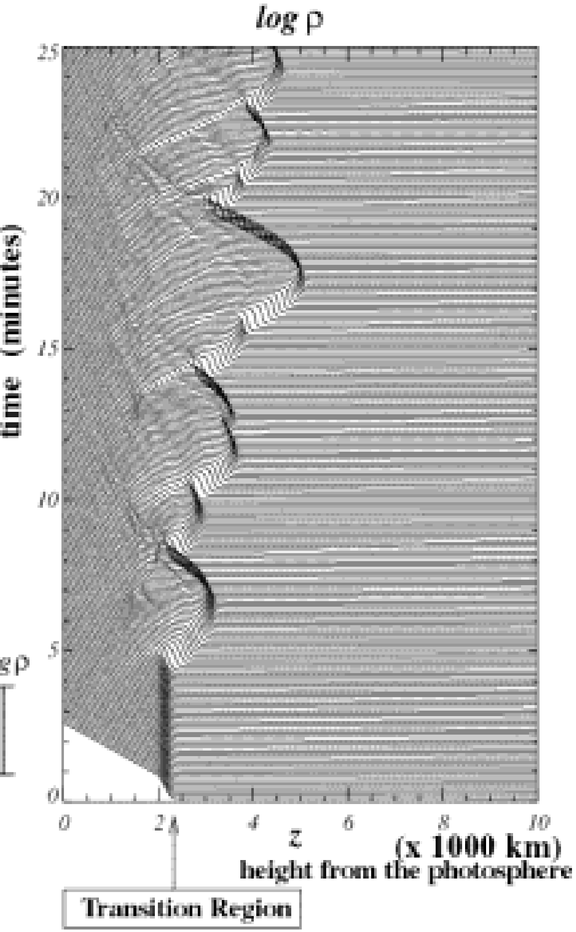
Solar spicule
(supersonic jet in the solar chromosphere)

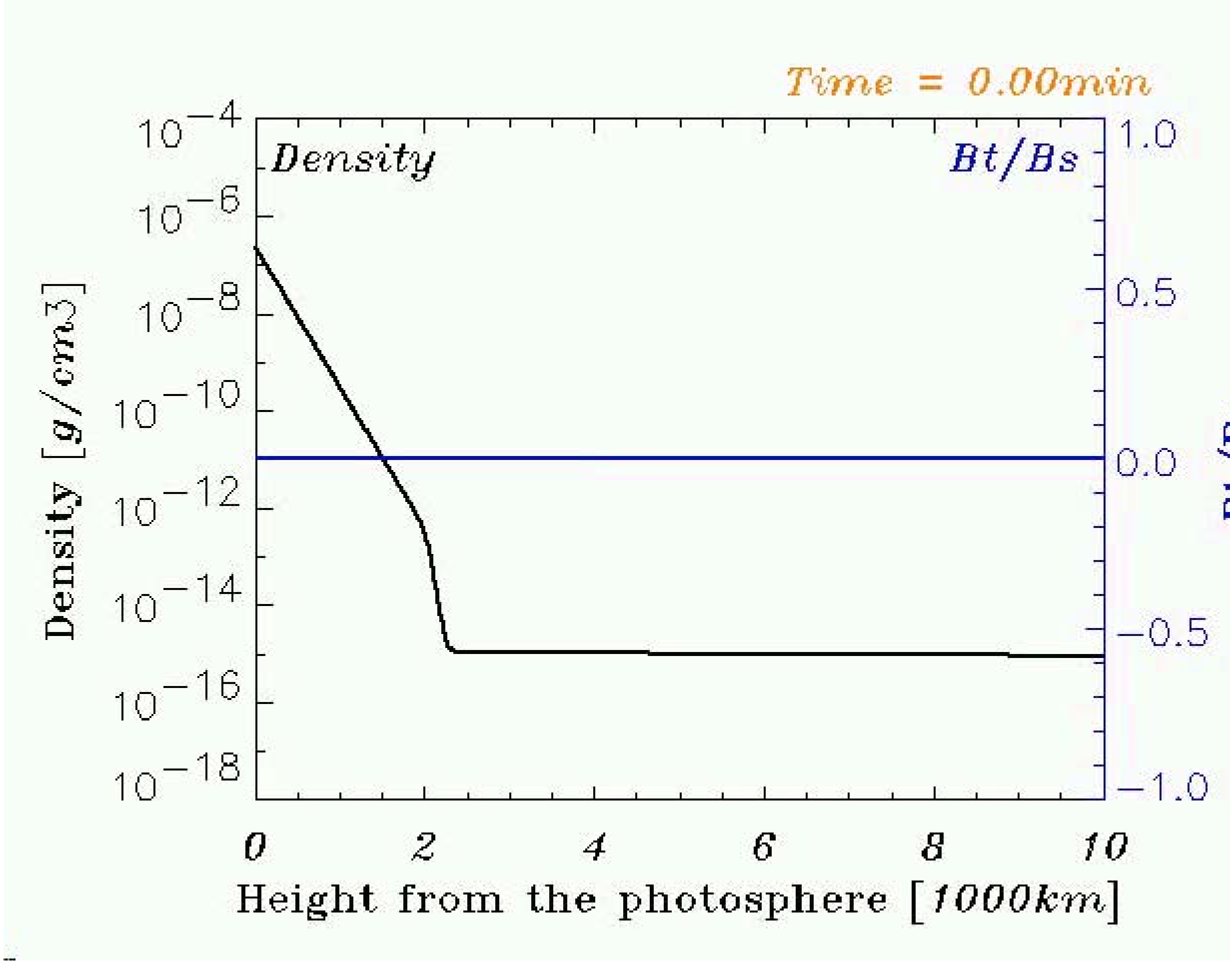




Alfven wave model of spicules:

numerical simulation (Kudoh-Shibata 1999)





４. Examples of astrophysical MHD simulations

- Movies are interesting !
- Let’s enjoy these movies

１) magnetic reconnection model of solar flares and jets (Shimizu, Miyagoshi)

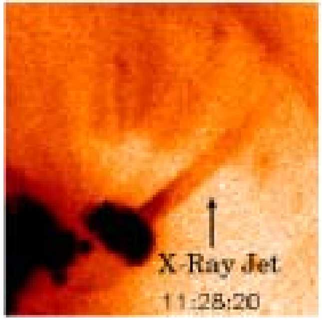
2) magnetic reconnection model of protostellar flares and jets (Uehara)

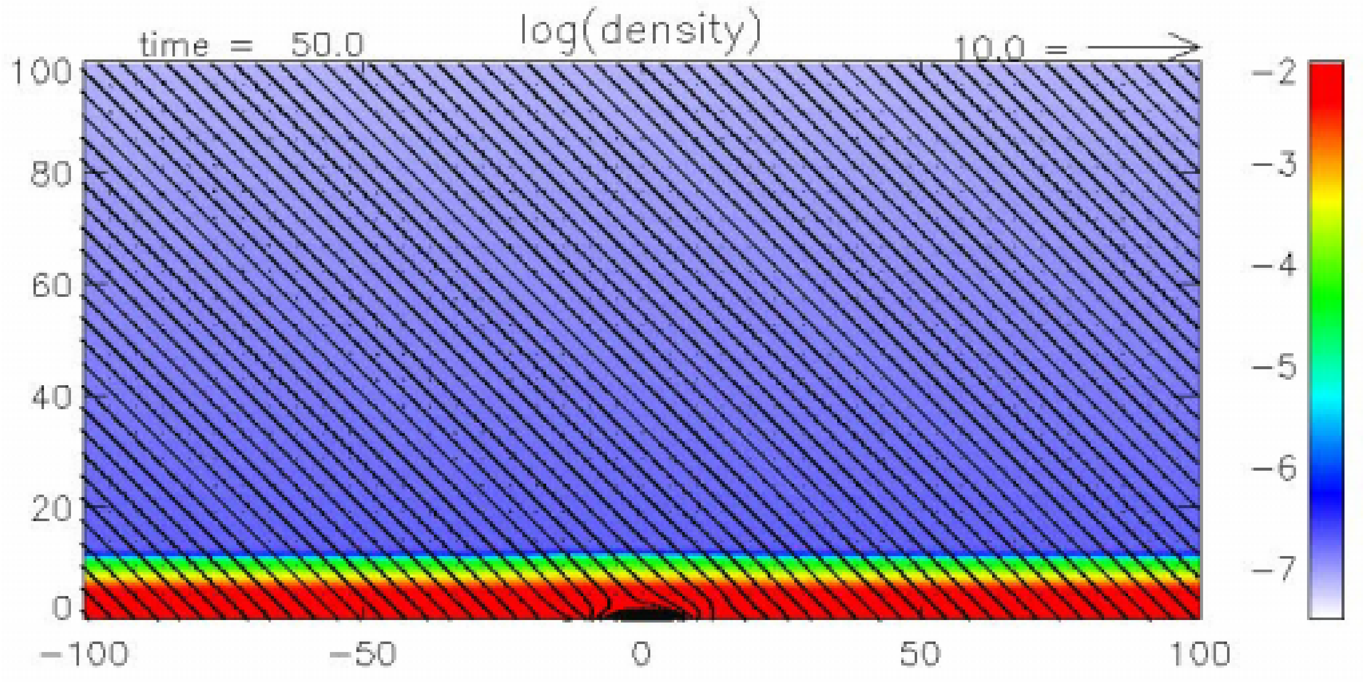
３) MHD model of astrophysical jets and collapsar (Kudoh, Mizuno)

MHD simulations of Solar coronal jets

anomalous resistivity model

$$\eta = \eta_0 (v_d - v_c)^2 / v_c^2 \qquad for \quad v_d > v_c$$
$$= 0 \qquad for \quad v_d < v_c$$
$$\text{where} \quad v_d = j / \rho$$

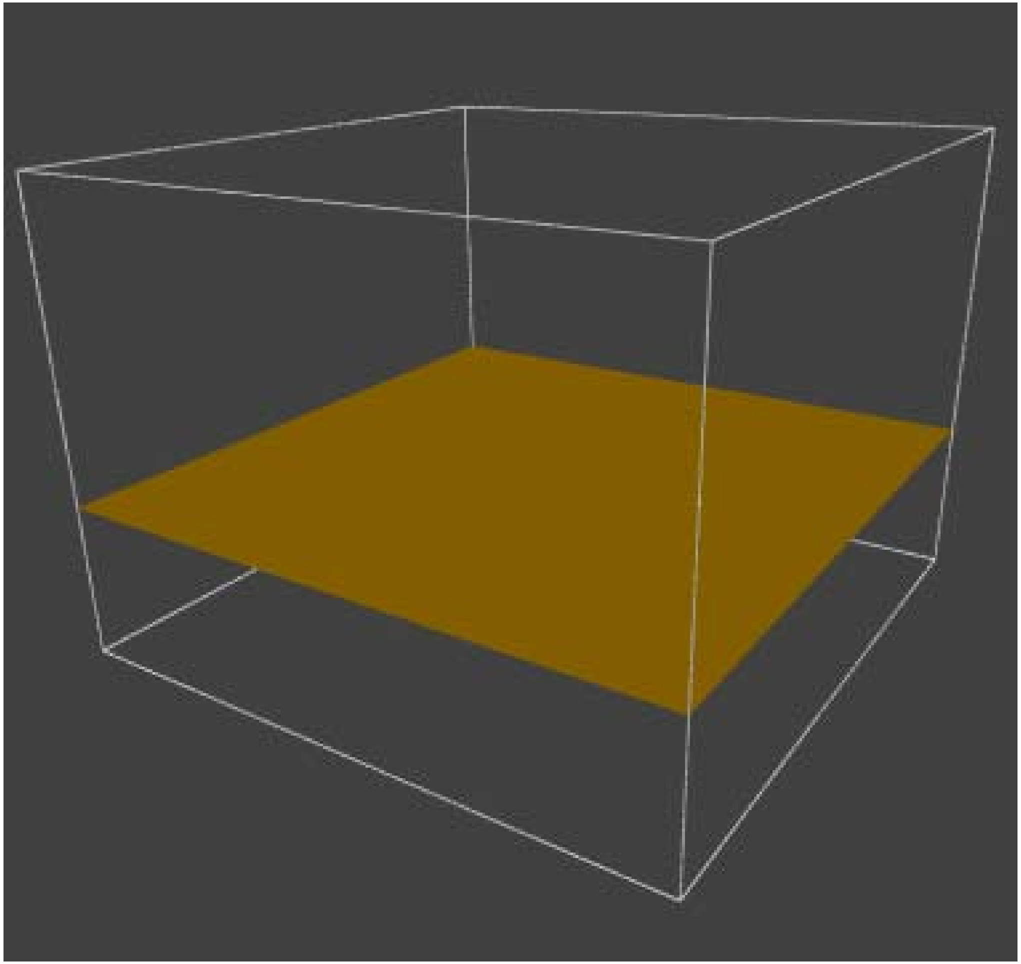


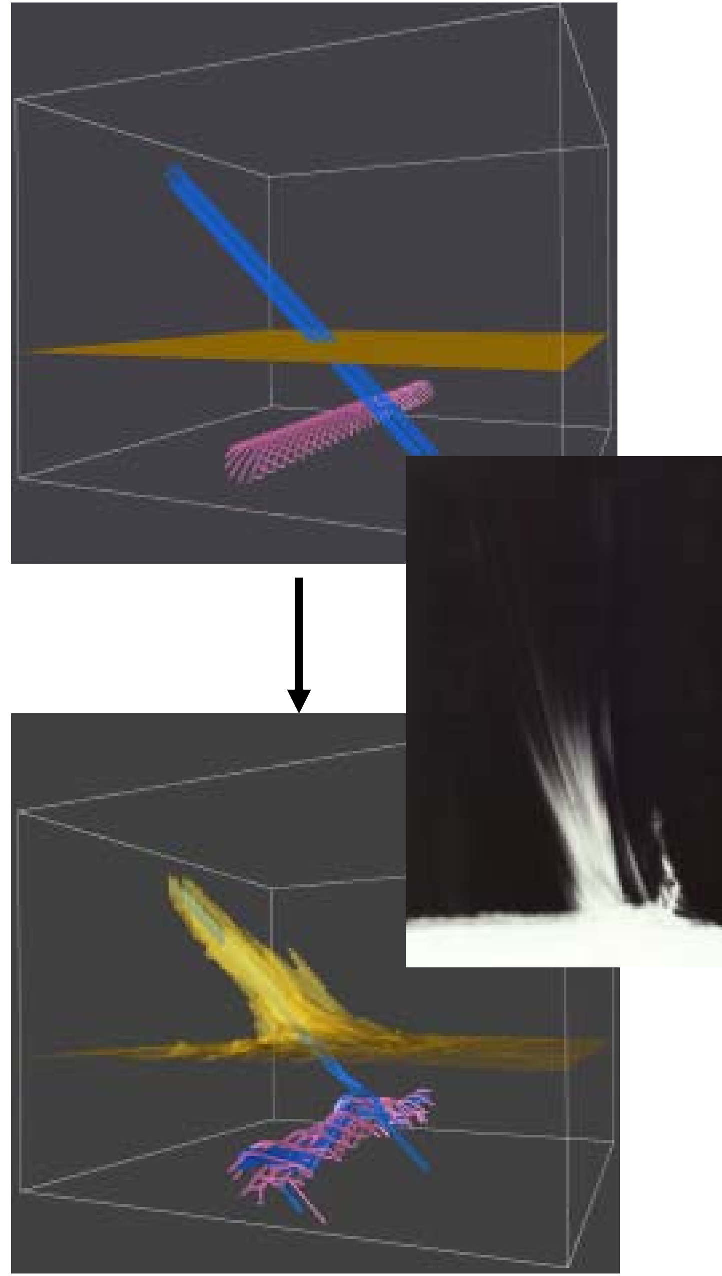


Solar coronal X-ray jet
(Yohkoh/SXT: Shibata et al. 1992,
Shimojo et al. 1996)

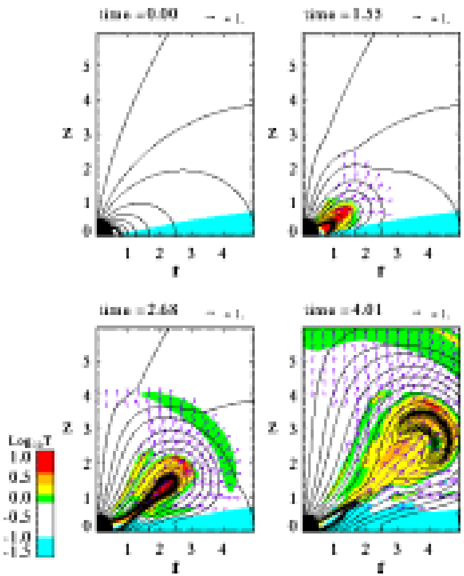
New simulations (Shimizu et al., 2006)
of MHD reconnection model of
Solar coronal jets
(Yokoyama and Shibata 1995, 1996)

3D-MHD simulation of jets (Miyagoshi et al. 2006)

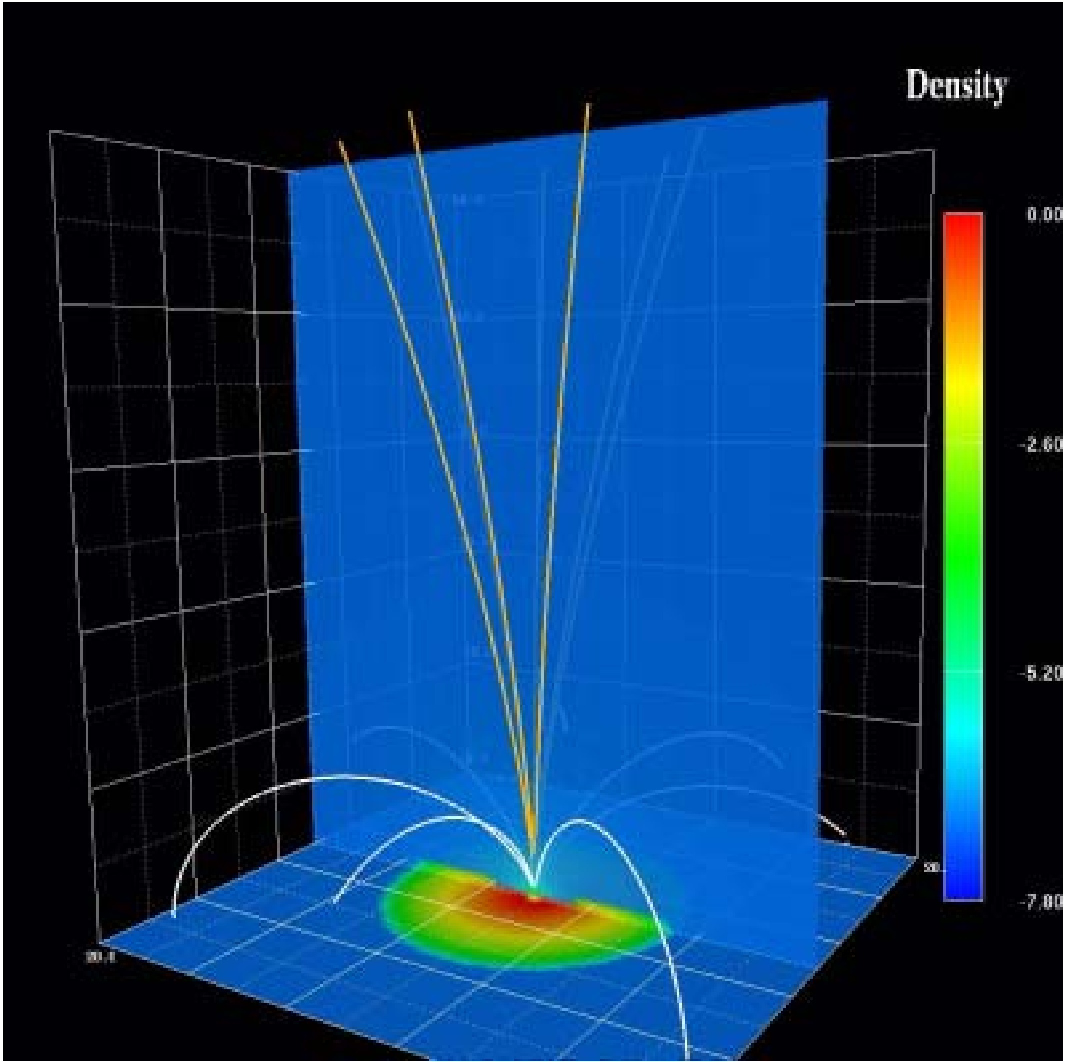




MHD model of protostellar jets as an extention of Hayashi et al (1996) model
(Uehara et al. 2006)



Hayashi et al (1996)

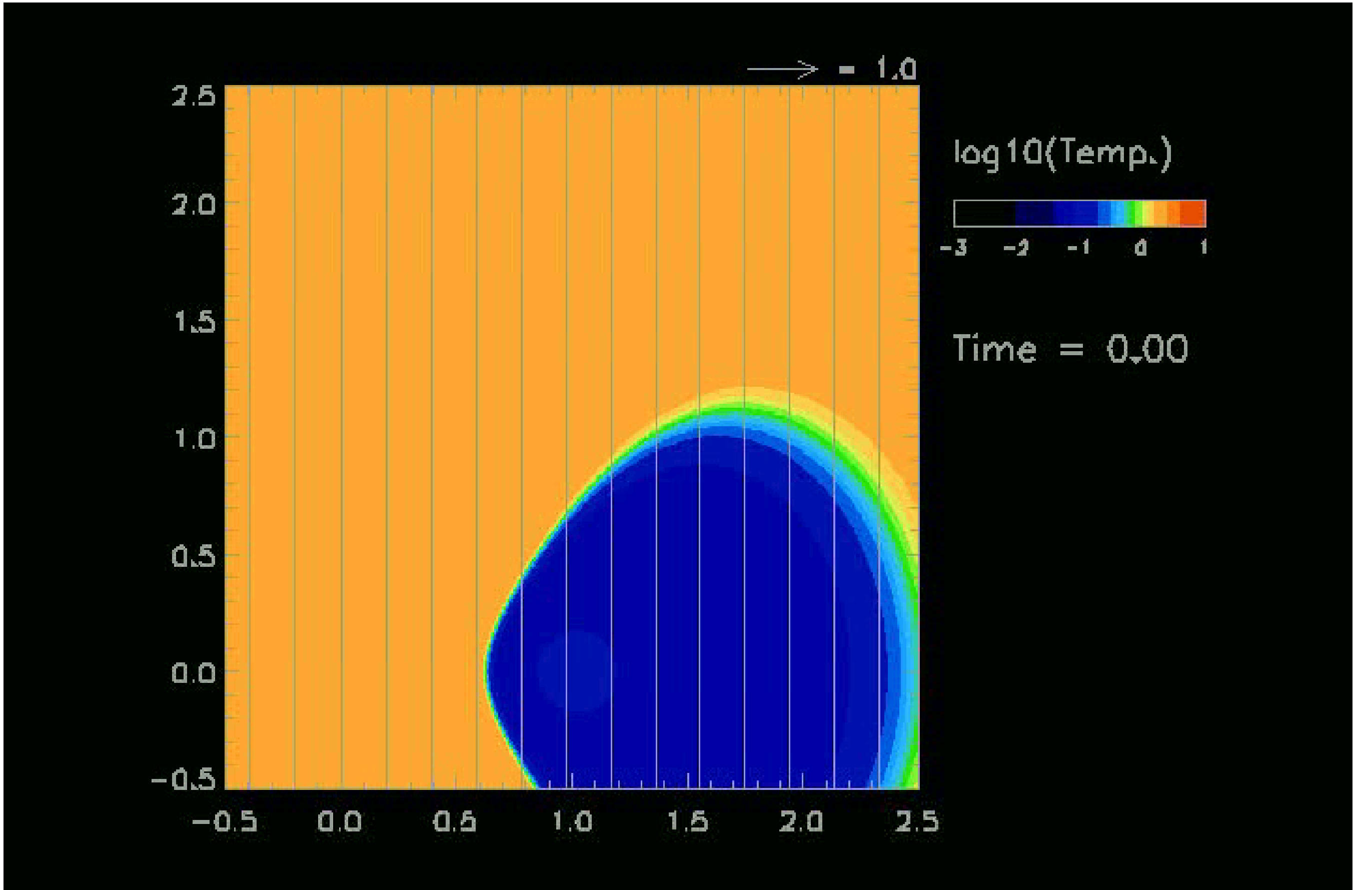


A jet consists of two component:
reconnection outflow and
disk wind

Numerical simulation of accretion disk

(Kudoh, Matsumoto, Shibata 2002, PASJ)

Magnetorotational Instability (Balbus and Hawley 1991) leads to turbulence and reconnection



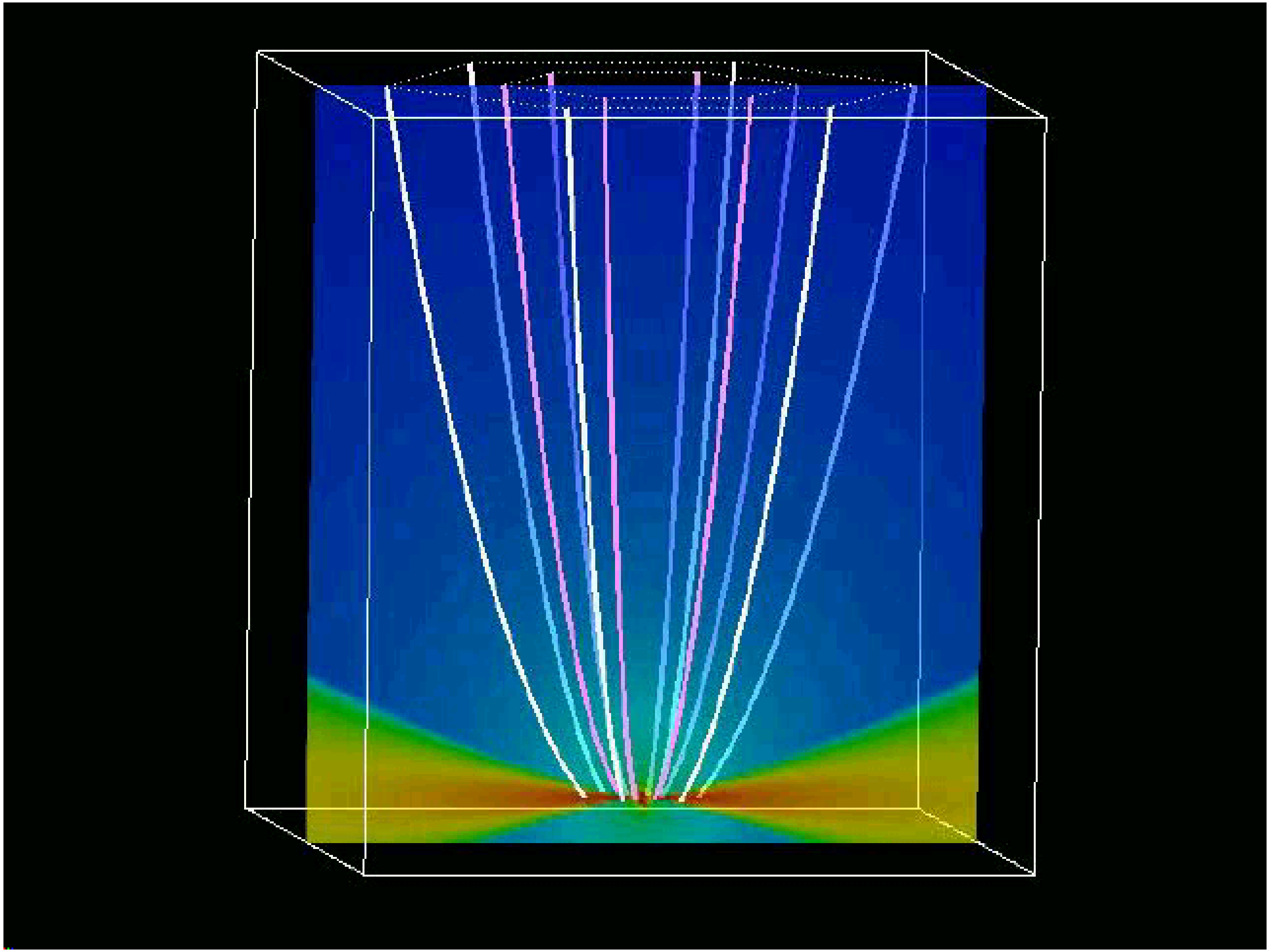
Cf)

Machida and Matsumoto 2005

Ibrahim et al 2006

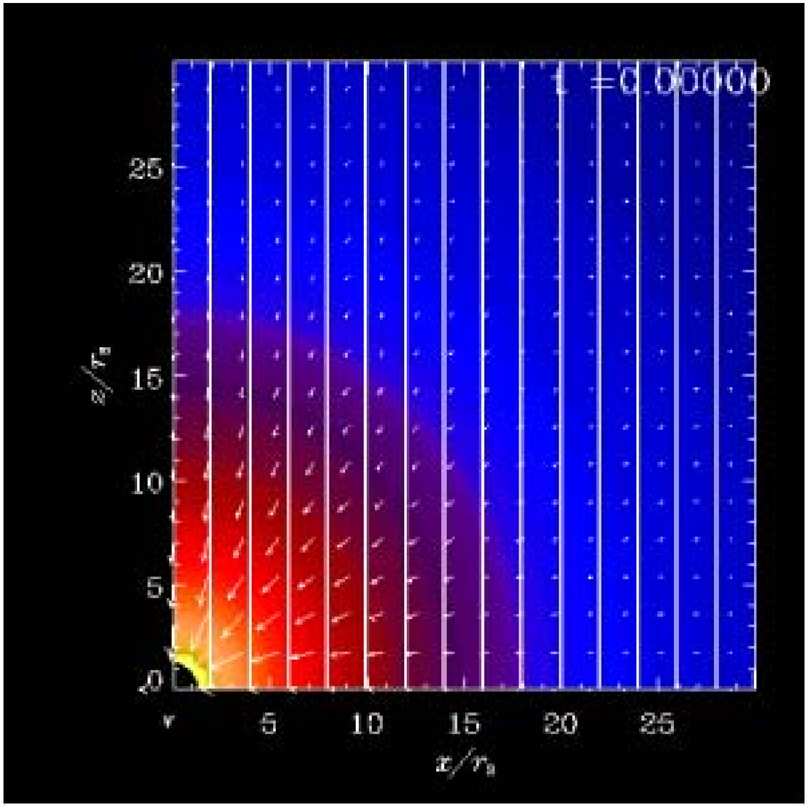
MHD model of astrophysical jets
(Kudoh et al　2006)

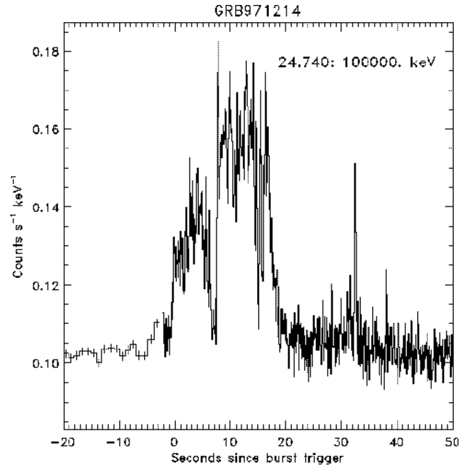
CIP-MOCCT scheme



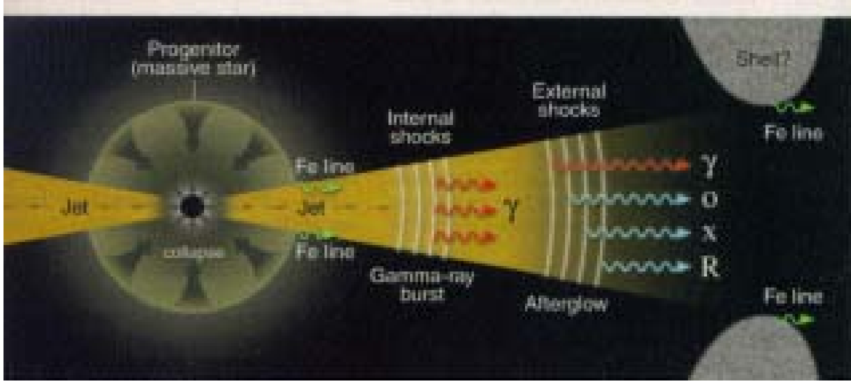
MHD simulation of collapsar as a model of gamma ray burst

Mizuno, Koide et al. (2004) ApJ 606, 395





Gamma
Ray burst
observatio

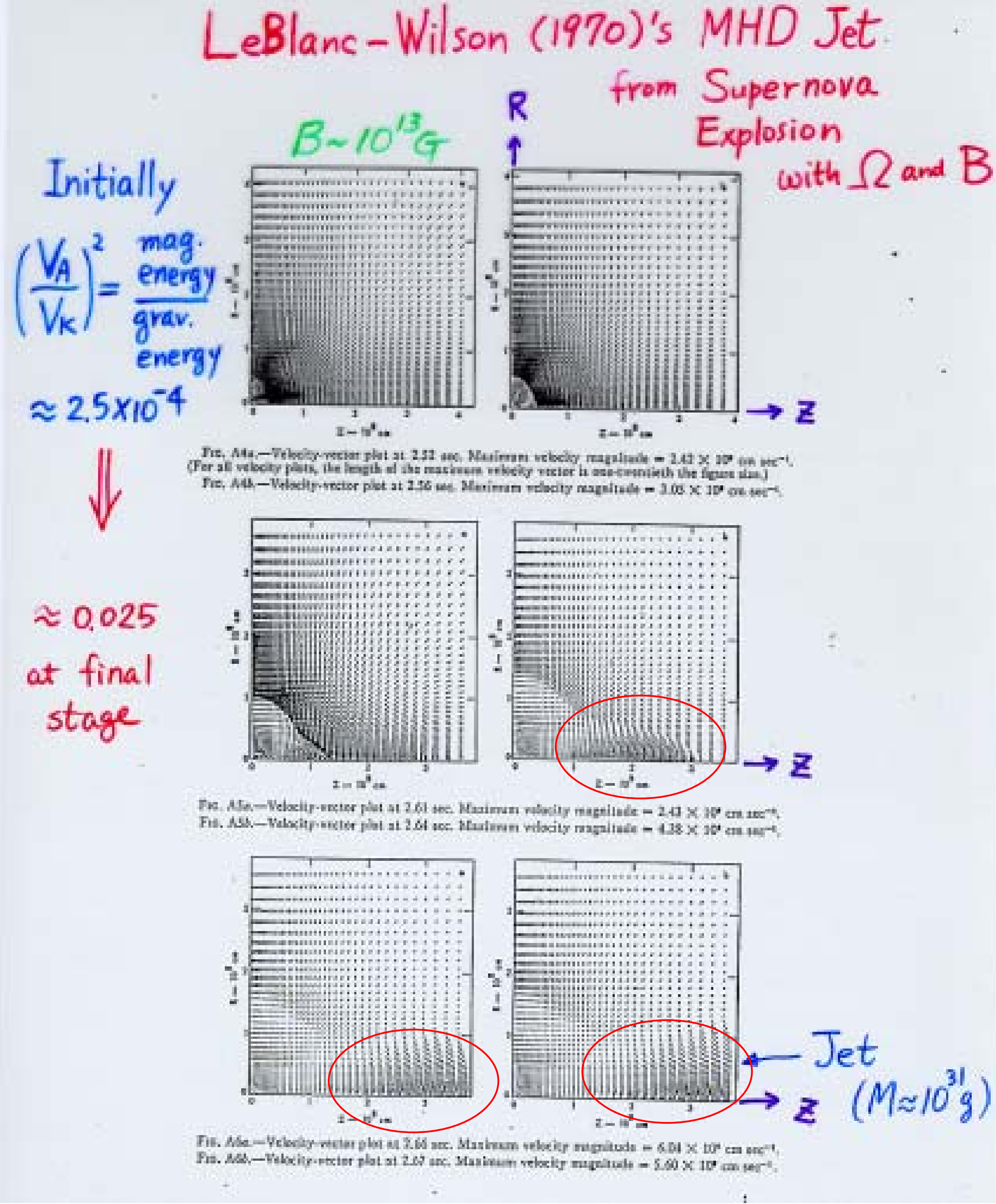


general relativistic MHD simulation with
Schwarzchild black hole Vjet ~ 0.2c

Collapsar model of
gamma ray bursts

Lebranc-Wilson (1970) MHD simulations of supernova collapsar

They thought that their simulations may be numerical artifact, so the results were presented only in Appendix



Merit of astrophysical MHD simulations

1) useful to understand qualitative properties of physical phenomena
2) enable astrophysical modeling, and play a role to bridge observations and theories
 (e.g., Yokoyama and Shibata 1995)
3) useful as a tool to discover a new phenomenon and physical rule.

Simulation is a numerical experiment.

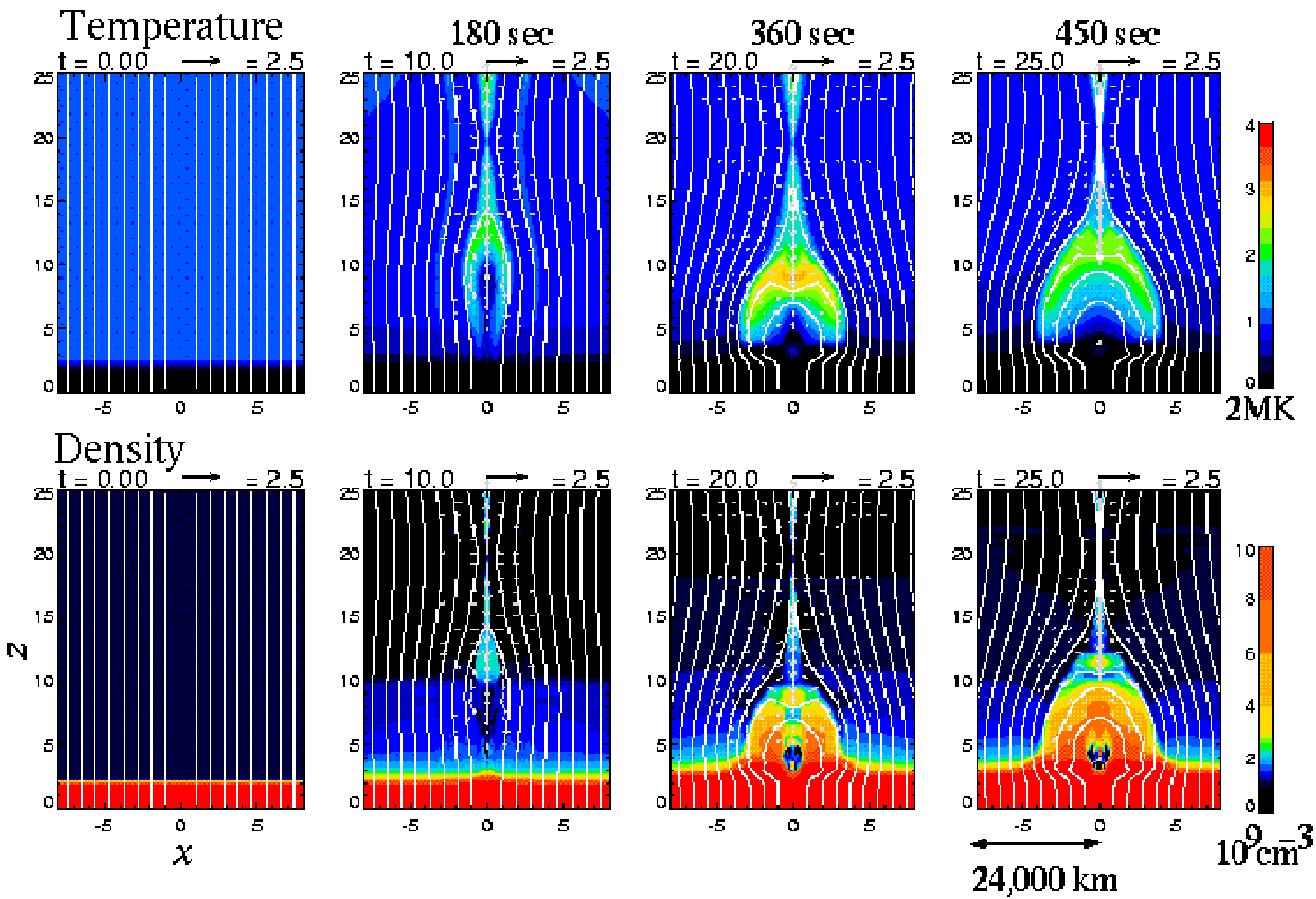
(ex scaling law by Yokoyama and Shibata 1998, spiral slow shocks by Shiota et al. 2005,

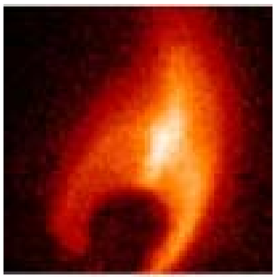
stability of reconnection solution by Hirose et al. 2004)

First self-consistent MHD simulation of reconnection including heat conduction and chromospheric evaporation

(Yokoyama-Shibata 1998)

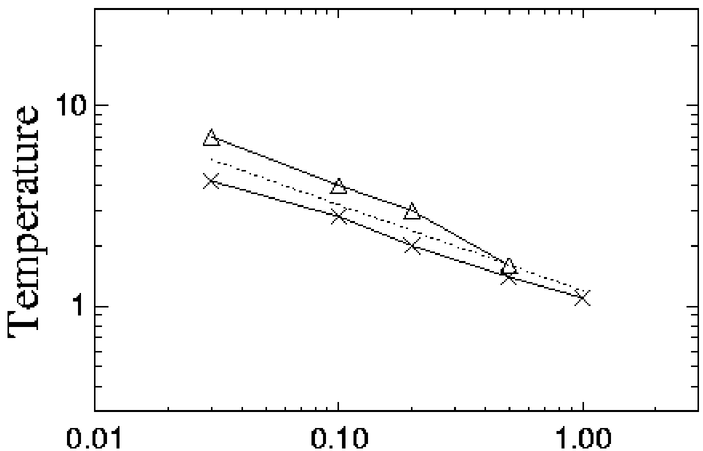
Lax-Wendroff＋implicit scheme



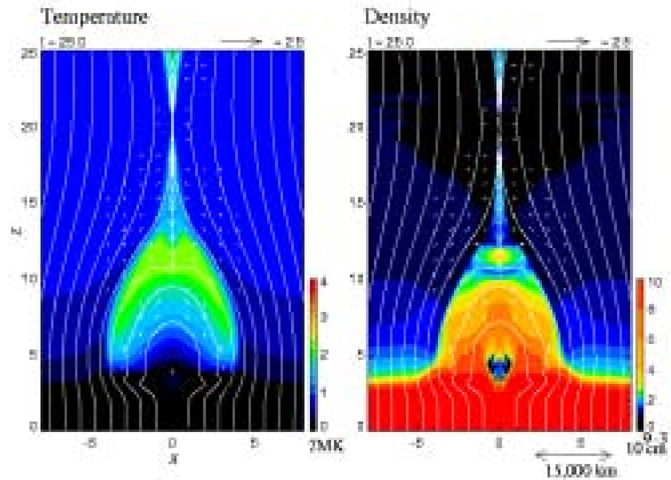


Solar flare
Observed by
Yohkoh soft
X-ray telescop

Flare temperature scaling law
(Yokoyama and Shibata 1998, 2001)



$$\beta = 8\pi p / B^2$$



$$T \propto B^{6/7} L^{2/7}$$

What determines the flare temperature ?

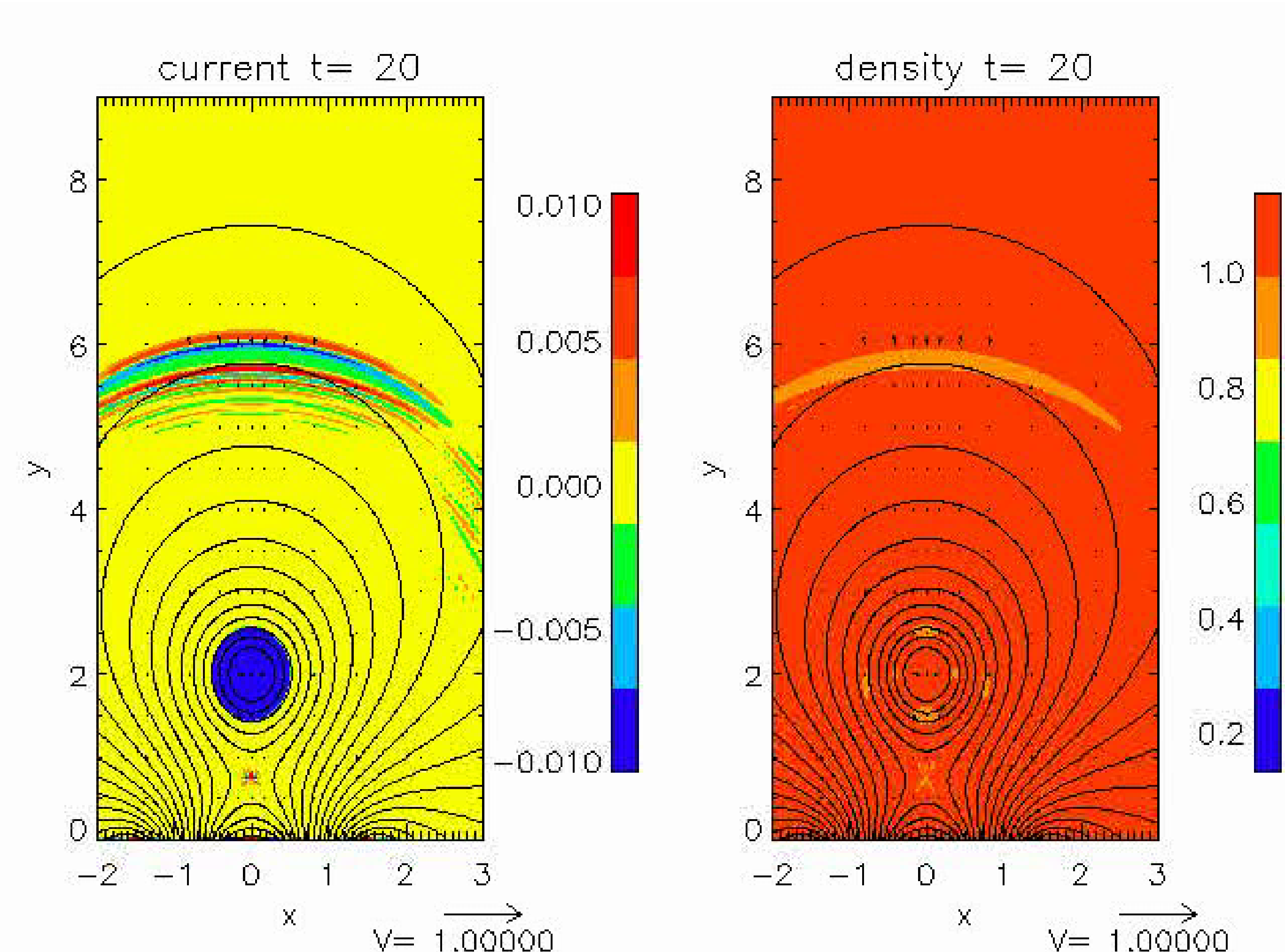
- Balance between reconnection heating and conduction cooling（Yokoyama and Shibata 1998, 2001)

$$B^2 V_A / 4\pi = \kappa T^{7/2} / 2L$$

$$\boxed{T \propto B^{6/7} L^{2/7}}$$

Discovery of Spiral Slow shocks associated with magnetic reconnection in coronal mass ejection model (Shiota et al. 2005, ApJ)

current density　　　　plasma density



Stability of exact reconnection solution (Hirose et al. 2004, ApJ)

The Astrophysical Journal, 610:1107–1116, 2004 August 1

Simulation subject in school of numerical astrophysics in 2003

NUMERICAL EXAMINATION OF THE STABILITY OF AN EXACT TWO-DIMENSIONAL SOLUTION FOR FLUX PILE-UP MAGNETIC RECONNECTION

Shigenobu Hirose
Department of Physics and Astronomy, Johns Hopkins University, Baltimore, MD 21218-268; shirose@pha.jhu.edu

Yuri E. Litvinenko
Institute for the Study of Earth, Oceans, and Space, University of New Hampshire, Durham, NH 03824-3525; yuri.litvinenko@unh.edu

Syuniti Tanuma and Kazunari Shibata
Kwasan and Hida Observatories, Kyoto University, Yamashina-ku, Kyoto 607-8471, Japan; tanuma@kwasan.kyoto-u.ac.jp, shibata(

Masaaki Takahashi
Department of Physics and Astronomy, Aichi University of Education, Kariya, Aichi 448-8542, Japan; takahasi@phyas.ai

Takayuki Tanigawa
Academia Sinica, Institute of Astronomy and Astrophysics, Taipei 106, Taiwan; tanigawa@asiaa.sinica.edu.tw

Takahiro Sasaqui
Department of Astronomy, University of Tokyo, Bunkyo-ku, Tokyo 113-0033, Japan; sasaqui@th.nao.ac.jp

Ayato Noro
Department of Physics, Chiba University, Inage-ku, Chiba 263-8522, Japan; noro@astro.s.chiba-u.ac.jp

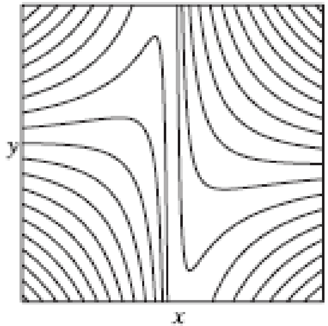
Kazuhiro Uehara
Department of Astronomy, Kyoto University, Sakyo-ku, Kyoto 606-8502, Japan; uehara@kwasan.kyoto-u.ac.jp

Kunio Takahashi
Department of Earth Science, Ibaraki University, Mito, Ibaraki 310-8512, Japan; kutaka@env.sci.ibaraki.ac.jp

Takashi Taniguchi
Department of Physics, Tokyo University of Science, Shinjuku, Tokyo 162-8601, Japan; taka0825-0612@mail.goo.ne.jp

and

Yuliya A. Terekhova
Institute for the Study of Earth, Oceans, and Space, University of New Hampshire, Durham, NH 03824-3525; yuliya.terekhova@unh.edu
Received 2003 September 1; accepted 2004 April 14



field lines

5. Dangers in astrophysical MHD simulations

- One can construct attractive simulation movies based on astrophysical MHD simulations, and those movies are loved and appreciated by many people. Hence one often forget to analyze the data in detail, and may become lazy in developing a new theory and writing scientific paper.

=> We have to forbid simulations for some period to complete science and paper.

Danges in astrophysical MHD simulations

- MHD simulations are generally difficult, so one tend to try to improve their code everyday, and forget to do astrophysics.

=> Unless you are a good researcher of numerical hydrodynamics, you should concentrate on astrophysics, and must write a paper at some point of research.

6. Summary

- Though astrophysical MHD simulations are not easy, there are a number of important puzzles remained, e.g., supernovae, solar/stellar flares, astrophysical jets, gamma-ray bursts, dynamo, galaxy formation, star/planet formation, etc.
- One common difficulty in these astrophysical problems is that there are huge dynamic range in space, time, and physical conditions. So numerical simulations treating multi-scale coupling is urgent and important direction for future.
-

Example of Future direction : simulations of mult-scale coupling

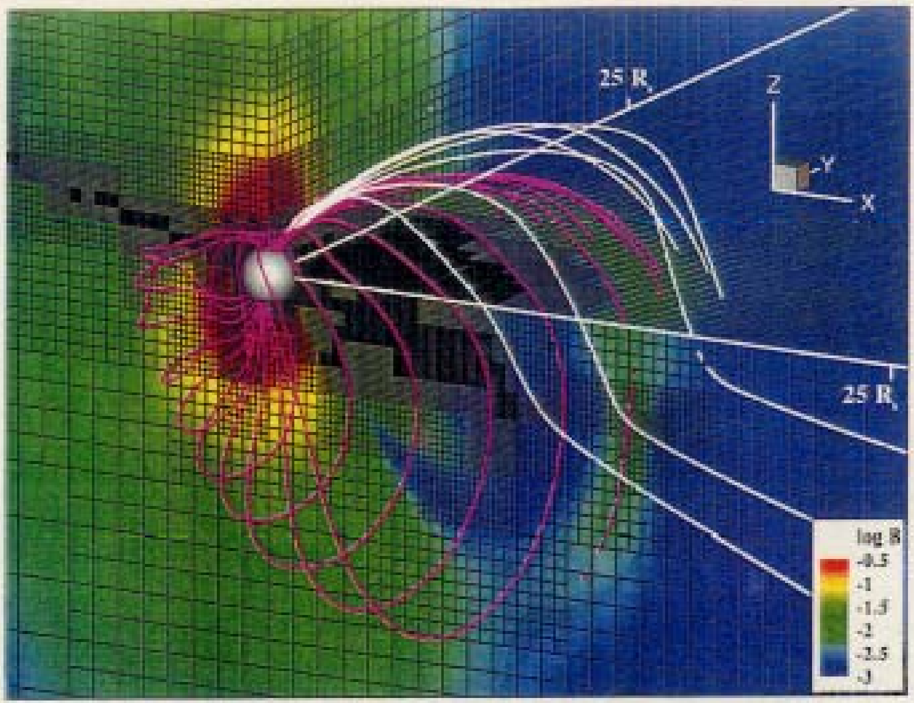


Plate 7. Three-dimensional configuration of magnetic field lines 9 hours after the initiation of the CME. The color shading represents $\log(B)$ in the (x, z) and (x, y) planes. The black lines are the computational mesh, white lines are open magnetic field lines, and magenta lines represent magnetic field lines with both ends connected to the Sun.

6. Summary

- Though astrophysical MHD simulations are not easy, there are a lot of important puzzles remained, e.g., supernovae, solar/stellar flares, astrophysical jets, gamma-ray bursts, dynamo, galaxy formation, star/planet formation, etc.
- One common difficulty in these astrophysical problems is that there are huge dynamic range in space, time, and physical conditions. So numerical simulations treating multi-scale coupling is urgent and important direction for future.
- Let’s challenge these puzzles !